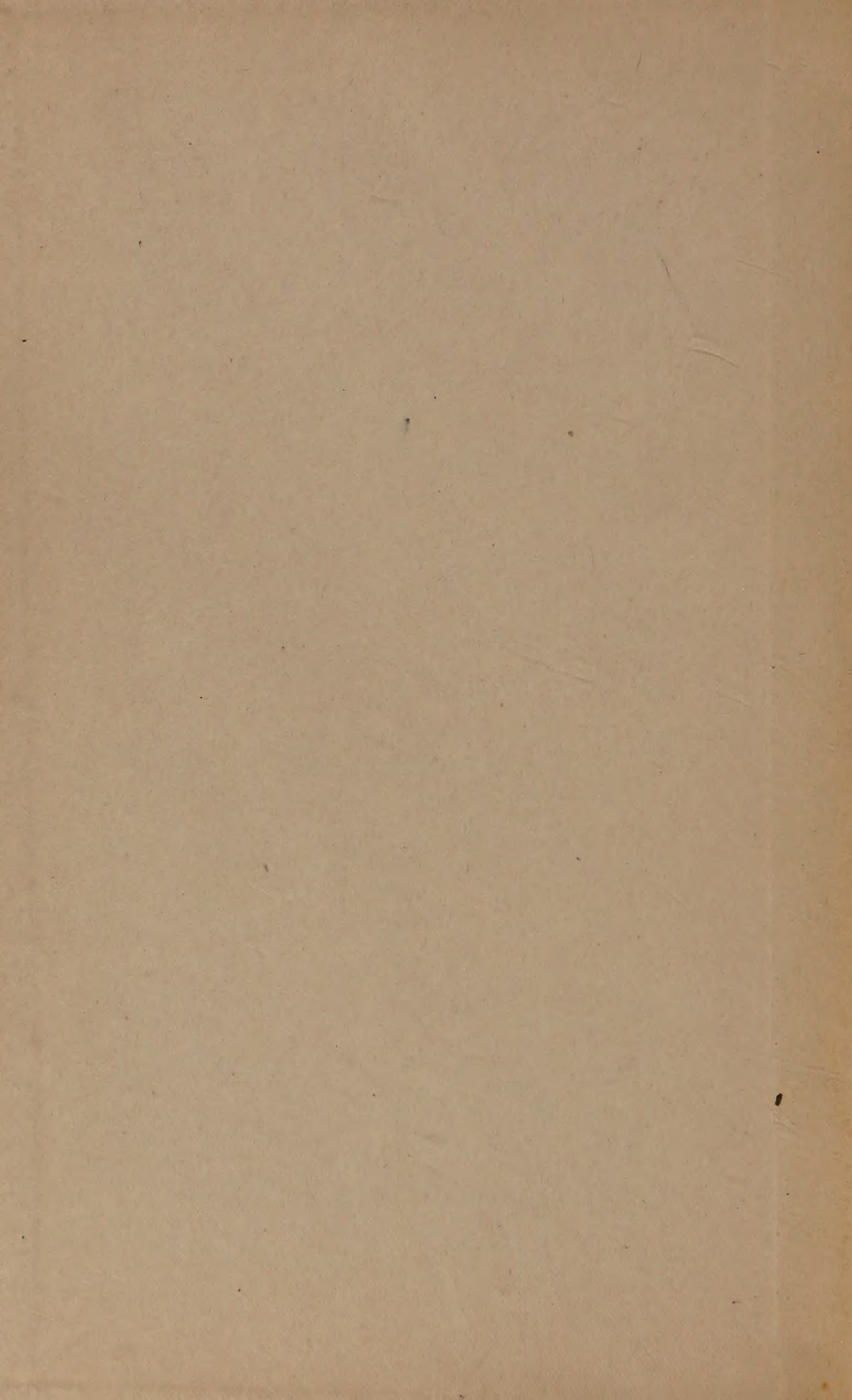
















# GRAPHICAL ANALYSIS







# GRAPHICAL ANALYSIS

of

STRESSES INVOLVED IN DESIGNING  
FRAME STRUCTURES

*INTENDED PRIMARILY FOR STUDENTS  
OF ENGINEERING*

BY

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G. P. SCHUBERT

## PREFACE

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The course of instruction outlined in this book is intended to give the student a working knowledge of some of the fundamental principles involved in the design of frame structures.

While it is understood that the stresses induced in any structure by the external loading may be determined mathematically, it is here intended, that on account of the relative simplicity, that these stresses be determined by means of graphical analysis. It may be said that the most common and efficient types of frame structures are built up of parts or members which are designed to withstand certain stresses in tension or in compression, or both.

In order to make safe and economical designs it is necessary to know first the kind of materials available, and when that is known, to make an analysis of the project to determine accurately the nature and extent of the stresses involved. The analysis begins with the determination of the requirements of the structure; then, with the available type of building materials in mind, proper distribution is made of the loads to be supported; lastly, the exact stress in each of the parts or members of the structure is found so that they in turn may be designed to meet those requirements in the most efficient manner.

The calculations necessary for finding the exact sizes and types of these members, together with the proper size and type of the joint connections, will be left for investiga-

tion in other courses. The entire effort in this course will be confined to an attempt to arrive at the theoretic value of the stresses that exist under different conditions in the members of the various frame structures.

The work of the student in the drawing room will consist of making accurate pencil drawings as solutions of the various types of frame structures illustrated on the problem sheets of the Appendix A.

A different set of conditions will be assigned to each member of the class, and with these he will be expected to make accurate graphical analyses of the problems in question.

In order to get the full benefit of his efforts, he must study all of the possibilities of his solved problem and make comparisons with similar types of structures to learn if possible where his design might have been changed so as to make it more economical and efficient. The comparison of the different types of trusses which might be used for the same purpose can be made with ease, and the change produced in any or all of the stresses in any frame by a change or modification of a few of its members can be readily shown.

The series of problems given in the appendix will lead the student by regular steps from the simple force triangle, which is the basis of this system, to a number of more complicated solutions which may be applied to the various structures for the purpose of determining the stresses therein.

Since this, like many other books, is intended for students of engineering, a word of advice directed especially to them may not be inappropriate. In order to gain a clear understanding of all engineering problems, it will be necessary



for the student to make certain numerical computations and sketches. In this work a neat and systematic method should be cultivated. The practice of performing computations on any loose scraps of paper that happen to be at hand should at once be discontinued by every student who has followed it, and he should, hereafter, solve his problems in a special book provided for that purpose, accompanying them by such explanatory remarks as may seem necessary to render the solution clear. Such a note book, written in ink, and containing the fully worked out solutions and sketches of the examples and problems suggested in this book and in the class room, will prove of great value to every student who makes it.

Before beginning the solution of any problem a sketch diagram should be drawn whenever it is possible, for a diagram helps the student to clearly understand the problem, and a problem thoroughly understood is half solved. A mental estimate of the final result, if made just before the actual solution, will also prove beneficial.

The text books used and the class notes which the student makes while in college will probably become obsolete in a sense, but will ever remain to him as the most vivid explanation of some of the fundamental principles of engineering.



# CONTENTS.

| Art.          | Page |
|---------------|------|
| Preface. .... | 3    |

## PART I.

### CHAPTER I.

|                                      |    |
|--------------------------------------|----|
| 1. General principles involved. .... | 6  |
| 2. Definitions of terms used. ....   | 10 |
| 3. Outline of methods. ....          | 12 |

### CHAPTER II.

|                                                        |    |
|--------------------------------------------------------|----|
| 4. Drawing and general instructions. ....              | 15 |
| 5. The engineers' scale. ....                          | 16 |
| 6. The triangle of forces and system of notation. .... | 17 |
| 7. Stresses in the frame. ....                         | 20 |
| 8. External forces on the frame. ....                  | 21 |
| 9. Examples to illustrate general principles. ....     | 24 |

## PART II.

### CHAPTER III.

|                                                      |    |
|------------------------------------------------------|----|
| 10. Roof trusses. ....                               | 29 |
| 11. Types of roof trusses. ....                      | 29 |
| 12. Loads on roof trusses. ....                      | 33 |
| 13. Example analysis of stress in roof trusses. .... | 39 |

### CHAPTER IV.

|                                                 |    |
|-------------------------------------------------|----|
| 14. Solution by symmetry. ....                  | 49 |
| 15. Trusses with interior loading. ....         | 52 |
| 16. Theory of moments. ....                     | 55 |
| 17. Trapezoidal shaped and bridge trusses. .... | 57 |
| 18. The swing or drawbridge. ....               | 65 |
| 19. The Scissor truss. ....                     | 66 |
| 20. The two-hinged arch Prob. 29. ....          | 69 |
| 21. The three-hinged arch Prob. 30. ....        | 69 |
| 22. Solution by reversal of the diagonal. ....  | 73 |
| 23. Solution by trial. ....                     | 75 |

|                                              |    |
|----------------------------------------------|----|
| 24. The Hammer-beam truss. ....              | 76 |
| 25. Supporting bridge and roof trusses. .... | 81 |

## CHAPTER V.

|                                                     |     |
|-----------------------------------------------------|-----|
| 26. Action of the wind. ....                        | 87  |
| 27. Intensity of wind pressure on roofs. ....       | 88  |
| 28. The effect of wind pressure on the truss. ....  | 92  |
| 29. Computing the wind load on the roof truss. .... | 92  |
| 30. Computing the reactions for wind load. ....     | 93  |
| 31. Stress diagrams for wind loads. ....            | 94  |
| 32. Tabulation of results obtained. ....            | 97  |
| 33. Lateral or sway bracing. ....                   | 98  |
| 34. The portal. ....                                | 100 |
| 35. Trusses supported by columns Prob. 42. ....     | 103 |

## CHAPTER VI.

|                                                    |     |
|----------------------------------------------------|-----|
| 36. Frames with inclined loads "Headframes". ....  | 112 |
| 37. The sand wheel. ....                           | 119 |
| 38. The sheave wheel. ....                         | 122 |
| 39. Winding drums. ....                            | 125 |
| 40. Trusses with interior loads. ....              | 125 |
| 41. Graphical analysis of stresses in joints. .... | 126 |

## APPENDIX A.

|                                                                                             |                                   |
|---------------------------------------------------------------------------------------------|-----------------------------------|
| Frame diagrams and data on problems to be solved by the<br>students in the drawing room.... | 129, 130, 131, 132, 133, 134, 135 |
| Index .....                                                                                 | 136, 137                          |



# PART I.

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## CHAPTER I.

### ARTICLE I. GENERAL PRINCIPLES.

Graphical analysis of stress is a study, by means of diagrams, of the stability and equilibrium of a structure, the relations that exist between the applied loads or external forces, and the stresses caused by their existence; it is also a study showing how the wind or any oblique force may alter the amount of stress arising from the fixed external forces. It can be shown that the graphical method of analysis may be used to determine the extent of the stresses existing in some systems of trussing which otherwise appear insoluble.

The use of graphical analysis for the solution of problems of construction has of late years become very widespread. Representing to the eye the forces which exist in the several parts of a frame possesses many advantages over their determination by calculation.

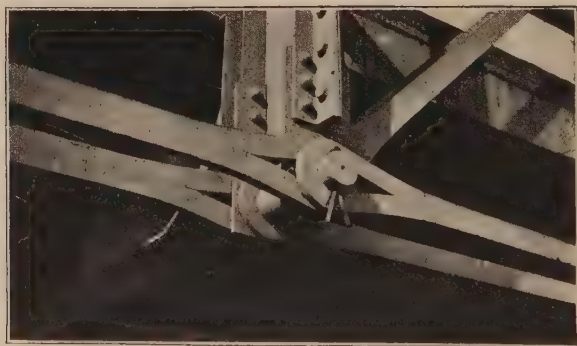
The accuracy with which the figure is drawn depends upon the ability of the draftsman, but may be readily tested by numerous checks. Anyone who has mastered the few general principles will soon discover for himself that it is a simple and direct means of analysis and that those who prefer arithmetical computation will find the stress diagram to be a useful check on the calculation.

Founded on a principle absolutely correct, the diagrams drawn according to these methods give results depending for their accuracy on the exactness with which the lines have been drawn and on the scale by which they are measured. With ordinary care the forces may be obtained much more accurately than the several parts of the frame can be proportioned.

## ARTICLE 2. DEFINITIONS OF TERMS.

A **FORCE**, in graphical statics, is understood to be a weight or pressure applied in a certain direction and at a particular point. It may be either an external, internal, concurrent or non-concurrent force. In analyzing the stresses in a structure, it is customary to resolve the forces into a common plane and then to make the remaining determinations in that plane.

AN **EXTERNAL FORCE** on any frame structure may be either a load, such as a snow load, wind load or weight of materials of construction; or it may be the reaction required to balance or support these loads.



**Fig. 1.**

A Pin-joint. Note the Eye-bars, Pin, Strut or Post, Diagonal Deck-bracing and Deck-beams.

An INTERNAL FORCE or stress is created in a member of the structure, and since that structure, for the purpose of this analysis, is assumed to have pin joints, Fig. 1, the stress will be either tension or compression and its line of action will coincide with the lateral axis of that member.

CONCURRENT FORCES are those forces which meet at a common point. For example see Fig. 2.

EQUILIBRIUM is that state of rest in which the external forces act at such points and to such an extent that there is no tendency for the frame to move either laterally, vertically or in rotation.

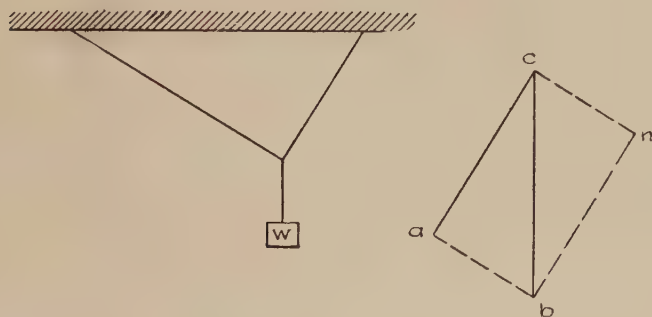


Fig. 2.

#### NEWTON'S LAWS:—

(1) All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted upon by some external force that causes a change.

(2) A force acting on a body in motion or at rest produces the same effect whether it acts alone or with other forces.

(3) To every action there is always a reaction in the opposite direction.

A MEMBER of a trussed structure is assumed to have no weight, but to be able to withstand either tensile or compressive stress applied axially.

A PIN JOINT is one that allows the members meeting at that joint to swing freely in the one plane in which these members lie. The pin-joint is actually used in the construction of certain frame structures, but for the purpose of making the graphical analysis of a structure we assume *all* of the joints to be pin-joints. See Fig. 1.

A FRAME is a rigid structure made up of one or more triangular panels so arranged that when the lengths of the sides of these panels are fixed, the frame will be rigid.

A STRESS DIAGRAM is a graphical representation of the stresses that exist in the several members of the frame or truss.

A TRUSS is a structure used to support loads; its members are so arranged that each is subjected to a stress only in the direction of its length; that is, in tension or in compression. The points where the members of the truss are joined are called the joints. The simplest form of truss is triangular in shape.

### ARTICLE 3. OUTLINE OF METHODS.

CONDITIONS: In the structures about to be considered, attention is called to the fact that in the actual construction of the same, members will be designed either for tension or for compression or both, depending upon the various strains which they may be called upon to resist. The joints may be rigid or of the pin variety; but in the theoretical solution following, we must work on three assumptions: (1) that all of the frames are made up of members capable of resisting tension or compression equally well;

(2) that the members of the frames themselves have no weight. (Weight of the members will be considered in a later article); (3) that all of the joints are pin-joints. With these conditions fixed, it follows, that in order to have a rigid truss or structure, the frames must be made up of one or more panels, each of which is a triangle. The only exception is the case in which we have a symmetrical frame together with symmetrical load, in which case it is sometimes possible to have a trapezoidal-shaped panel and at the same time to have a stable truss. See problem 12 and 21 of the Appendix.

**REPRESENTATION OF FORCES:** A force is said to be fully determined when its magnitude, direction, and point of application are known. In dealing with problems in the statics of rigid bodies, the magnitude, direction, and line of action of a force are the elements commonly involved. The equilibrium or motion of such a body is not affected by transferring the point of application of a force to any other point in the line of action.

**THE RESULTANT OF ANY SYSTEM OF FORCES LYING IN THE SAME PLANES:** The magnitude and direction of the resultant of any system of forces lying in the same plane may be found geometrically by representing the given forces by the sides of a triangle or polygon, taken in order. The closing side in reverse order is the resultant in magnitude and in direction.

The line of action of the resultant may be found geometrically by means of the principle that the resultant of two forces lying in the same plane must pass through their point of intersection. In case the magnitude of the resultant is zero, the forces either form a couple or are in equilibrium. If the resultant is a couple, its moment may



be found algebraically, or the given forces may be combined geometrically into a single resultant couple. Equilibrium is the special case of a couple whose moment is zero. If the lines of action of the given forces intersect at a common point, the line of action of the resultant will pass through that point, its magnitude and direction being found by the methods previously stated.

The conditions for equilibrium are as follows: (1) The sum of the moments (See art. 16) must equal zero. (2) The summation of all vertical components of forces must equal zero. (3) The summation of all horizontal components of forces must equal zero. (4) Where there are three non-parallel forces, they must, when produced, intersect at a common point.

With the above outlined methods of procedure, with fixed condition such as pin-joints, and inelastic members, and with the frame in equilibrium, we are prepared to proceed with the making of a graphical solution of the problem in statics as explained by example 1. in Article 6.

## CHAPTER II.

### ARTICLE 4. DRAWING AND GENERAL INSTRUCTIONS.

Before beginning with the actual graphical solution of any problem it is well to point out the following: Since the accuracy of the final result depends directly upon the accuracy with which the student lays out his work, great care should be exercised in drawing all lines, and all triangles and straight-edges should be tested to see that they have true edges and that the angles are actually  $30^{\circ}$ ,  $45^{\circ}$ , or  $60^{\circ}$  as the case may be. The "T" square should not be used in the usual manner unless the edge of the drawing board has been tested and is known to be perfectly smooth and true. The lines should be drawn with a hard pencil, 6H or 8H, having a sharp round point which may be kept sharp by using a file or a piece of sandpaper. The objection to a *chisel*-pointed pencil for this work, although it does usually give a sharp clean-cut line, is that it is liable to cut-out from the straight edge without detected.

In general, letter both drawings of the frames and of the stress diagrams and mark the value and kind of stress on each of the members of the frame and on the reactions. Use a plus sign to indicate when a member is in compression and a minus sign to indicate when a member is in tension. Answers to problems having more than one set of results are to be tabulated as per form given with that problem. Number each problem and indicate the scale used, both for the frame and for the stress diagram. Give all data for each problem such as load, span etc. Make an effort to get

your problem on a sheet in a neat and systematic order and avoid soiling the sheet by keeping the triangles, straight-edge and scales clean.

It is advisable to draw the figure of the frame to quite a large scale as the lines of the stress diagram are drawn parallel to the several members of the frame. If anybody objects to the fact that a slight deviation from the exact direction will materially change the length of some of the lines, and therefore give erroneous results, it may be suggested that just so much change in the form of the frame will produce a proportional change in the forces. One is therefore warned so that due allowance for such deformation may be made by the proper distribution of material.

#### ARTICLE 5. THE ENGINEERS SCALE.

The Engineers' or Surveyors' chain scale, conveniently named decimal scale, is the proper kind to use in the solutions of problems by means of graphical analysis. It consists essentially of units, such as inches, divided into 10, 20, 30, 40, 50 or 60 parts for every unit or inch on the scale. The most convenient kind of scale for all-around work is the triangular type having six different scales, two on each edge. For example, the edge marked 20 may be used in such manner that for every pound of stress to be represented to scale on any stress diagram, a distance of  $\frac{1}{20}$  of an inch or one division of the scale will be used. If the load is large,  $\frac{1}{20}$  of an inch may be made to represent 10, 100, or 1000 pounds, in which case the scale of the drawing will be designated as 200, 2000 or 20,000 pounds per inch.

The scale which is chosen for any particular problem should be one which, when used, will finally give a stress diagram somewhat larger than the frame diagram itself.

When the proper scale has been chosen, the load line drawn, and the entire stress diagram completed, the values of the stresses in the different members of the frame may be read directly from the scale without any arithmetical computation whatsoever, the only necessity being to estimate the values of the fractions of divisions on the scale. Under NO conditions should any scale be used such as 1 inch equal 184 pounds, or 1 inch equals  $588\frac{1}{2}$  pounds, but instead use a scale of 1 inch equals 200 pounds and 1 inch equals 600 pounds, these being the nearest even decimal scales. The metal scale guard or clip will be found helpful as its use prevents error due to reading of the wrong scale.

#### ARTICLE 6. TRIANGLE OF FORCES AND NOTATION.

Taking it for granted that if two forces acting at a common point are represented in length and in direction by the two adjacent sides  $ca$  and  $cn$  of the parallelogram Fig. 2, then the resultant will be equal to the diagonal  $cb$  of the figure, drawn from the same point, and it follows that a force equal to the resultant, and acting in the opposite direction, will balance the first two forces. Hence, considering one-half of the parallelogram, we have the well-known proposition that, if three forces in equilibrium act at a single point, and if the triangle be drawn with the sides parallel to these three forces, these sides will be proportional in length to the forces. The forces will also be found to act in order around the triangle and must necessarily lie in the same plane. If the magnitude of one force is known, the other two can be readily determined. Having one or more forces about a point known in magnitude and direction, two "unknowns" may be solved for by this method. (By "unknowns" we mean either the magnitude of

two forces whose lines of action are known, or one force that is unknown both in magnitude and in direction.)

A SYSTEM OF NOTATION will here be introduced which will be found useful and convenient when applied for the purpose of determining the nature of the stress in any member of a truss or frame. In the frame diagram Fig. 3, write a

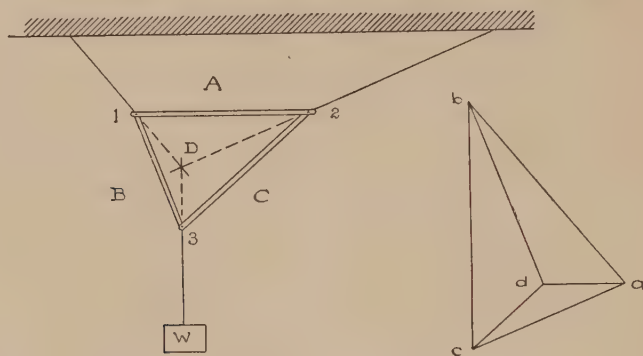


Fig. 3.

capital letter in every space which is cut off from the rest of the figure by lines, real or imaginary, (not construction lines), along which forces act. Then *D* represents the space within the triangular frame, *A* the space limited by the external forces acting at 1 and 2 and *B* the space between the line to 2 and the line which carries the weight. Then let that piece or part of the frame or that force which lies between any two letters be designated by those two letters, thus: the upper base of the triangle is *AD*, the right-hand bar is *DC*, the cord to the point 1 is *AB*, and that to the weight, or the weight itself is *BC*, etc.

In stress diagrams drawn to determine the magnitude, direction and nature of the several forces acting upon or in the frame, the corresponding small letters will be used; then *bc* will be the vertical line representing the force in



$BC$ ;  $ab$  the tension in the cord  $AB$ ; and  $ac$  the pull or tension in the cord  $AC$ .

The student will here note that, in the following argument, the several members are named in one general fixed direction when reading the "name" from the frame, and that it is afterwards represented in the stress diagram as acting in a given fixed direction depending upon the order of the lettering or naming that it receives from the frame. For the sake of uniformity of method, always read in the same rotative order about a joint; say, counterclockwise. (Opposing the direction of the hands of a clock).

Returning to Fig. 3, let us suppose that a rigid triangular frame is made fast to the cords by attaching them to the vertices of the triangle, while their directions are undisturbed. It is evident that the same stresses still exist in these cords, if the frame has no weight, and that the portion of the cords within the triangle may be cut away without destroying the equilibrium of the combination. Hence we see that the equilibrium of the frame is assured so long as the directions of the cords or *forces external to the frame meet, if prolonged, at a common point*.

The external forces  $BC$ ,  $CA$ , and  $AB$  taken in the order  $BCA$ , or that of passing around the exterior of the triangle in the direction contrary to the movements of the hands of a clock "counterclockwise", gives the triangle of forces  $bca$ , in which  $bc$  acting in a known direction, i. e., downwards, determines the direction of  $ca$  and  $ab$  in relation to their points of application to the frame, since, for equilibrium, Art. 2, they must follow one another in order around the stress triangle.

## ARTICLE 7. STRESSES IN THE FRAME.

The left hand apex, 1, of the triangle Fig. 3 is in equilibrium under the combined action of the three forces  $AB$ ,  $BD$ , and  $DA$ . (reading the letters in the same rotative order as before.). Then, having found the direction and magnitude of the force  $AB$  in the previous section, and knowing the inclinations of the lines of action of the other two, it follows that the three forces meeting at this joint are represented by the three sides of the stress triangle as before.

Begin with  $AB$ , the fully known force, and pass from  $a$  to  $b$ , because that is the direction of the action of the force  $AB$  on the joint under consideration. Next; from  $b$ , draw  $bd$  parallel to  $BD$ , prolonging it until a line from its extremity  $d$ , parallel to the member  $DA$ , strikes or closes on  $a$ . The stress  $bd$  is found in  $BD$ , and stress  $da$  exist in  $DA$ . The direction in which we passed around  $abd$ , that is, from  $a$  to  $b$ , and then to  $d$ , shows that  $BD$  and  $DA$  both exert a tensile stress on the joint where they meet.

Next, take the lowest joint, remembering again to take the three forces in equilibrium in the same order in which the external forces were taken, and commencing with the first known one, pass, on the stress diagram, from  $d$  to  $b$ ; then, having just found that  $bd$  represents the pull of  $BD$  on the left hand apex of the frame, it will be seen that  $db$  must be the equal and opposite pull  $DB$  on the lowest joint. Next, pass downward along the line  $bc$ , the direction in which the load acts; and finally we draw  $cd$  from  $c$ , parallel to the member  $CD$ . If the construction of the figures and diagrams has been carefully executed, this last line will close on the point  $d$ , and the direction in which we pass over it, from  $c$  to  $d$ , shows that the member  $CD$  exerts a

tension on the lowest joint. If the student will now pass over the triangle *cad*, which must represent the right hand joint, he will see that the directions just given have been complied with.

If we can imagine Fig. 3 to be inverted, then the weight will bear down on the upper apex of the triangle, and we will find, upon drawing the stress diagram, that the three external forces are thrusts, and that compression exists in each member of the frame.

#### ART. 8. EXTERNAL FORCES ON THE FRAME.

In order to make these first principles more plain, let us take another case. Suppose a triangular frame Fig. 4

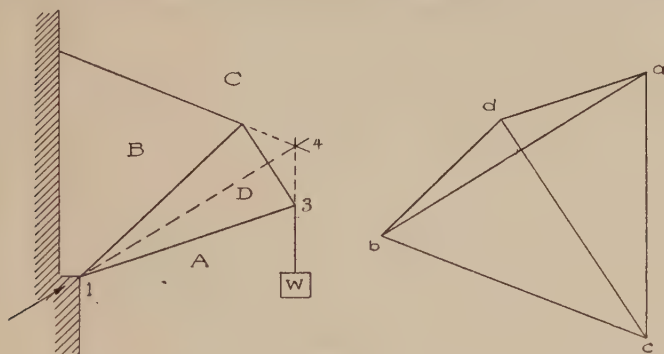
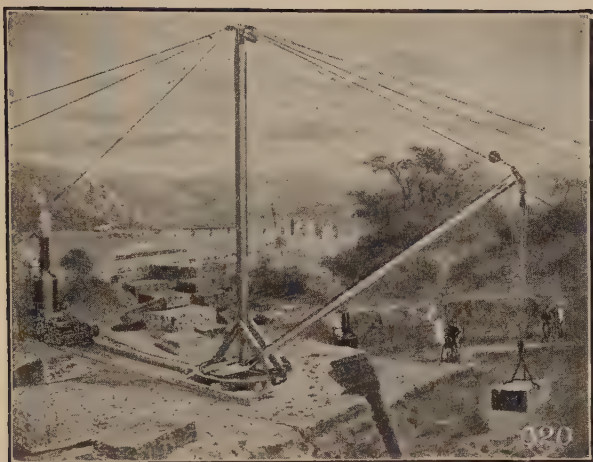


Fig. 4.

to rest against a wall by one corner, to have a known weight suspended from the outer corner, and to be sustained by a cord attached to the third corner and secured to the point 2. Since this frame is at rest under the combined action of three external forces which are not parallel, their lines of action must meet at a common point, and since the known direction of two of the forces, *AC* and *CB* will meet at 4 if prolonged, the force exerted on the frame by the wall at 1 must have the same direction as the line 1—4. Then the magnitude and kind of the two unknown external forces

may be found by the following construction and by observing the rules on interpretation already outlined. Draw  $ac$ , veritically downward, equal to the known weight and force  $AC$ ; next draw from  $c$ , a line parallel to the cord and force  $CB$  and prolong it until, from its extremity  $b$ , a line may be drawn parallel to  $BA$  to meet the point  $a$ . Since we pass from  $c$  to  $b$ , and from  $b$  to  $a$ ,  $CB$  must pull toward and  $BA$  thrust against the frame.

In order to determine the nature of the stress in any member of a frame, take whichever joint is most convenient, for instance, the joint of Fig. 4 where the weight is attached. Draw a line,  $ac$ , downward to represent the external force, and then, observing the order in which the triangle of external forces was drawn, draw  $cd$  parallel to  $CD$  and  $da$  parallel to  $DA$ . Since  $cd$ , in the triangle  $acd$  (made up of the forces  $ac$ ,  $cd$  and  $da$ ), must represent a force acting upwards,  $CD$  exerts a tension on this joint; and, similarly,  $da$  (not  $ad$ ) shows that  $DA$  thrusts against the same joint and is in compression and should therefore be marked with a plus sign. Now take the joint at 1. Here, the reaction, as before ascertained, is  $ba$ ; next comes  $ad$ , the thrust of the piece  $AD$  against this joint; and lastly  $db$  drawn parallel to  $DB$  to close on  $b$ , the point of beginning which shows that  $DB$  also thrusts this amount against the joint 1.



Lidgerwood Manufacturing Co.

**Fig. 5.**

Guy Derrick with Bull Wheel and Swinging Gear.



Lidgerwood Manufacturing Co.

**Fig. 6.**

Stiff-leg Derrick with Bull Wheel and Swinging Gear.



## ARTICLE 9. ILLUSTRATING GENERAL PRINCIPLES — PROB. 8.

Let the frame 1-2-3 of Fig. 7 support the weight  $W$  and in turn be supported by the members 1-4 and 2-4 ( $CA$  and  $BC$ ). If all are pin joints, then the only position of the frame, when it is in equilibrium, will be found when it is possible to produce  $AC$  and  $CB$  so that they meet at a common point vertically above the weight  $W$ . (The external forces must meet at a point in order to have equilibrium, as was assumed in the beginning.) Draw the stress dia-

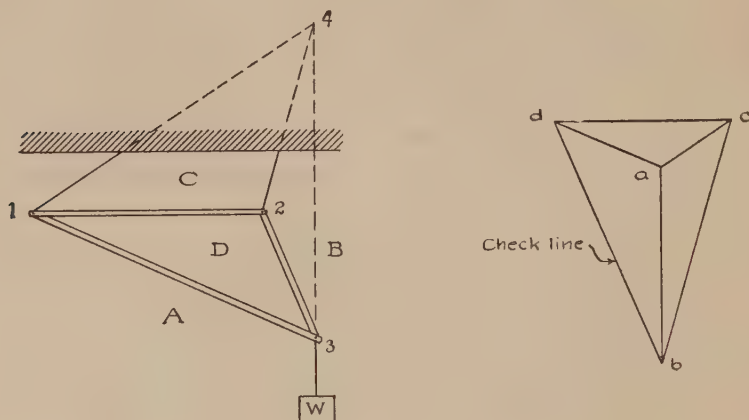


Fig. 7.

gram in proper order as follows:— Draw external forces first; draw  $ab$  downward and to scale equal to the known load  $W$ ; then draw a line  $bc$  through  $b$ . Since  $c$  cannot be definitely located except in that it will be somewhere on the line  $bc$ , pass on to the next external force  $CA$  and draw the corresponding line  $ca$  so that it will pass through the previously located point  $a$  and in this manner find the exact location of  $c$ , which must necessarily be at the intersection of these two lines. It will now be possible to begin and solve

either of the three joints, for in each case there are three forces, all having their lines of action fixed and one of them known in magnitude, so that the magnitudes of the other two (two unknowns) may be solved for about that point.

At joint 2 we have  $BC$  known; draw  $cd$  and  $bd$  to locate  $d$  and then, as a check, the line joining  $d$  to  $b$  should be parallel to the corresponding line or member  $DB$  of the frame.

The kind of stress (plus or minus) may now be determined by the usual method (Stresses in the frame. Art. 7), and its sign, together with its numerical value, written on the frame.

In mechanics, the polygon of force follows naturally from the triangle of force, it being simply a combination of several such triangles. The same rules will apply when we have to deal with several external forces or with a number of members meeting at one joint. (1) Draw the polygon of external forces for the whole frame, taking them in order around the truss, either to left or right, as may seem most convenient (say counterlockwise). (2) Take any joint where not more than two stresses are unknown, and draw the polygon of forces for it. Treat the members and external forces which meet at the joint in that same order (counterlockwise) in which the external forces were taken, and begin, if possible, with the first known force, so that the *two unknown forces will be the last two sides of that particular polygon or triangle*. (3) The direction in which any line is passed over, in going around the polygon as above directed, shows whether the stress in the member to which that line was drawn parallel, acts toward or from the joint to which the polygon belongs, and that stress is either compression or tension. The student must fully master

this principle in order to correctly interpret his diagram, for he will soon find that any method of determination by inspection becomes too complicated.

It is well to note the fact that every joint of the frame will be found represented in the stress diagram in the form of a triangle or polygon and that the stress diagram is reciprocal to the frame and vice versa.

Problem 9. serves to illustrate the relative stresses induced in the ordinary SHOP or JIB CRANE. This type of crane is often used for service in blacksmith and drill-shops and is capable of serving a floor area whose radius is equal to the length of the jib. The jib is generally an "I" beam which forms a track upon which a trolley with a chain-block may travel radially.

It may be readily seen that the greatest stress in the members of the crane will exist when the load is at the end of the jib. However, in order to give a better idea of the relative stresses in the frame without making several different stress diagrams, we will treat the problem as though there were a load on each joint along the jib. A vital point in the proper solution of the problem is the fact that while the mast of the crane is supported in a vertical position by the roof or by the upper floor of the building it will not be good design to place more than the necessary load upon this source of support. Therefore, the upper reaction is assumed to be horizontal, so that no vertical loading falls upon this upper floor. To carry out this assumption in the practical design, a pin in the end of the mast is allowed to project through a bearing or ring in the upper floor or roof.

Then, with a vertical construction line through the middle of the loaded jib to represent the center of gravity of all the loads, and a horizontal external force at the upper floor

support, we may find the intersection of the external forces and from this point and the point of application of the lower floor support determine the line of action of the third external force. With the external forces fixed, the remainder of the solution is simple and may be completed by following the methods outlined in the previous articles.

## QUESTIONS AND PROBLEMS.

1. Compare the stresses in the frame with the load on problem 4.
2. In problem 6, if the member *CA* were horizontal, would it be an advantage to remainder of the truss?
3. How does the frame of problem 7 differ from the actual frame of the arc-light crane?
4. Which position of the load on the jib-crane causes the greatest stress in the mast?
5. Sketch the stress diagram for problem 6 when a horizontal guy line exerts a pull of  $\frac{1}{2} W$  in order to "land" the load on a car or on a platform.
6. What is the triangle of forces?
7. In the jib crane, what is the usual direction of the top or upper supporting force? Why?
8. What should the elementary figures composing a truss be, in order to insure stability?
9. Explain a practical method for finding the character of of the stress in any member of a frame.
10. What kind of joints are frames assumed to have?
11. A gin-pole 60 feet long and weighing 1800 pounds is resting on a level piece of ground. It is proposed to raise it to a vertical position by means of a "block and tackle" attached to the top of the pole and "anchored" to the ground at a distance of 60 feet from the "butt" of the pole, and by raising the top of the pole a short distance above the ground. The pole is capable of resisting a 10 ton compressive stress together with any bending moment caused by its own weight. How high must the top of the pole be lifted above the ground in order to make it possible to lift it the remainder of the distance by means of the block and tackle?



## PART II.

### CHAPTER III.

#### ARTICLE 10. ROOF TRUSSES.

A roof truss is a structure built in a vertical plane and used primarily for the purpose of supporting the roofing material, which, in turn, serves to protect the contents of the building from the weather. It is supported by the side walls of the building. In providing such cover over a large floor space, it is desirable to avoid designs which include obstructions in the form of columns, partitions etc. The truss spans from wall to wall of the building and is placed at intervals that depend for length upon the length of the span and partly upon the arrangement of the window openings in the side walls or in the roof. The arrangement of

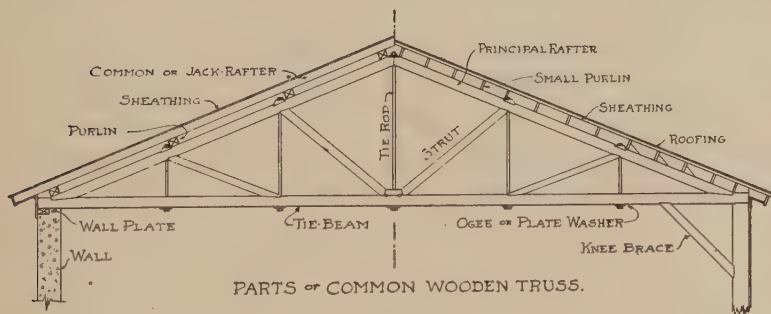


Fig. 8.

Above figure illustrates two different methods of supporting the roofing and gives the names of the different parts of a common wooden roof truss.

the several necessary parts of a common type of roof truss is illustrated by figures 8 and 9.

For stability, the elementary truss is triangular in shape, since that is the only polygon whose shape is not altered without altering the lengths of the sides. The larger trusses are built up of a series of such rigid elements and arranged, if possible, so that all external loads and reactions act only on the joints.

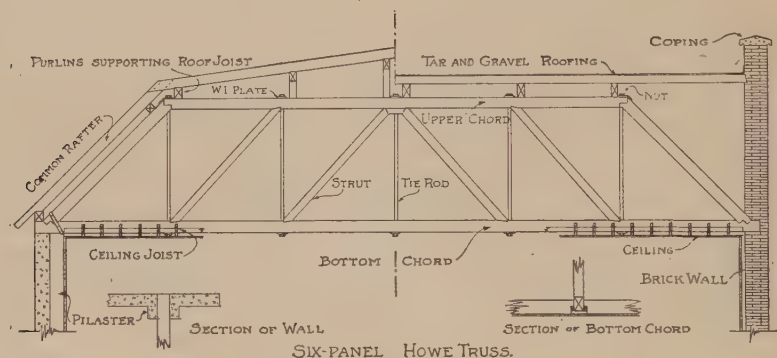


Fig. 9.

Illustrating the possibilities for using the Howe type truss in the construction of flat roofs and the manner of supporting the roof and ceiling loads.

The SPAN of a truss is the distance between the points of support. The points where the center lines of the adjacent members of a truss meet are called the JOINTS and, as here-inbefore mentioned, are assumed to be flexible in one plane or PIN JOINTS as illustrated in Fig. 1.

#### ARTICLE 11. TYPES OF ROOFS.

The roof, while it is intended primarily as a protection for the materials of construction and for contents of the building, should be given some attention when the building

is being designed, to see that it has a pleasing appearance as well as offering suitable cover for the space to be enclosed. There are several shapes or types or roofs used, their

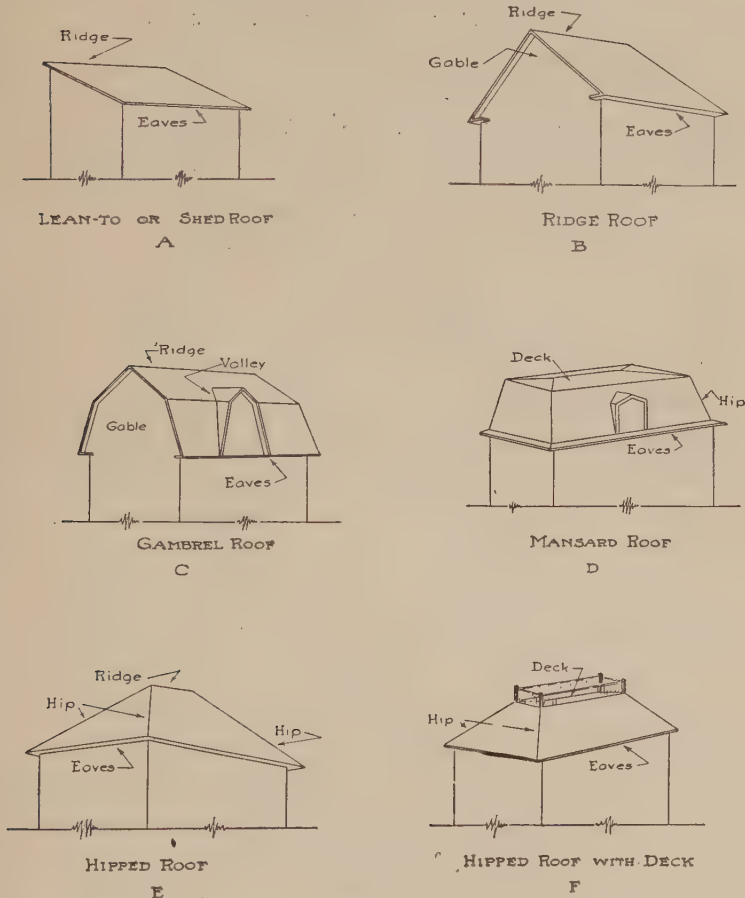


Fig. 10.

size, shape and cost varying as do their architectural appearances. Fig. 10 A, B, C, D, E, and F illustrate some of the more common types.

The lean-to or shed roof *A* is a type used in very ordinary construction and is easiest to build. The gable or pitch roof *B* lends itself to use in climates where snow is plentiful. It is the type of roof most used and perhaps the easiest to construct. It should be proportioned and designed so that the gables do not project too far over the face of the gable wall. On account of the possible entrance of snow and rain, it should have plenty of slope, not less than  $25^{\circ}$ .

The foregoing statements apply to the gambrel roof *C* as well, although a word of caution is necessary with regards to the shape of the gambrel. If improperly proportioned it will not present a pleasing effect. It may be given a more graceful appearance by adding a slight outward curve at the eaves.

The mansard roof *D* is similar to the gambrel roof except that it slopes on all four sides of the building and that the upper pitch is nearly flat. While shingles or similar roofing materials may be used on the steep pitch of this roof, other types of roofing must be used in providing suitable cover for the deck.

The hipped roof *E* which slopes at the ends or gables as well as the sides is another common type. Sometimes the end slopes begin higher up on the side walls, leaving a partial gable at the end of the building.

When this type of roof has a flat top it is called a deck roof (*F*), although it does resemble the mansard roof of figure *D*.

Besides these more common types there are some other structures which involve in their construction trusses listed in Appendix A. They are:—

9. Jib or Shop Crane. (Note effect of variable load).
10. Common triangular truss.

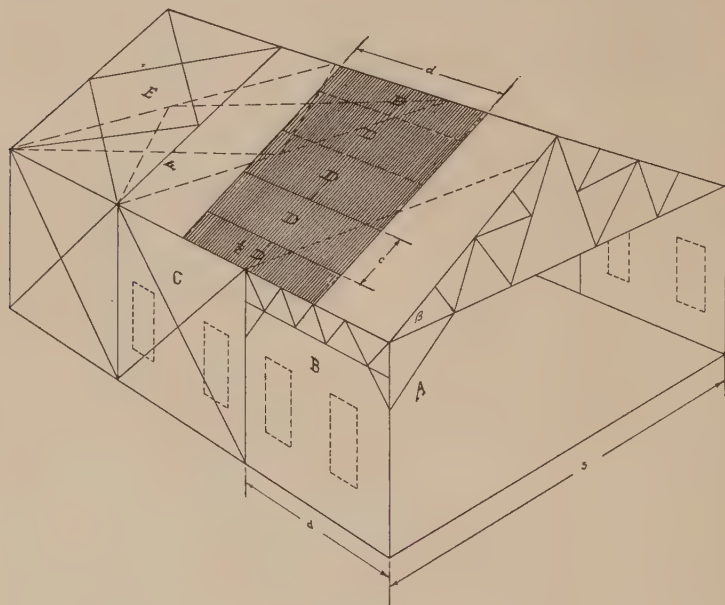
11. King-post truss.
12. Queen-post truss.
17. Fink truss. (Most common type of steel truss for roofs.)
18. Fink truss with camber.
21. Small trapezoidal truss.
22. Pratt truss, with tension diagonals, for bridges.
23. Howe truss, with compression diagonals, for bridges and roofs.
24. Warren girder, for bridges and flat roofs.
25. Bowstring truss, for highway bridges. See Fig. 28.
26. Parabolic bowstring truss, for highway bridges.
27. Typical single deck drawbridge.
28. Scissor truss. Short span and rigid joints.
29. Two-hinged arch.
30. Three-hinged arch.
35. Hammer-beam truss used mostly for churches.
36. Hammer-beam truss. Compare with problem 35.
38. Curb or mansard roof truss.
40. Truss for curved roof.
41. The Portal.
43. Simple head frame.
44. Sandwheel or Ferris wheel.
46. Sheave wheel.
47. Headframe with internal loading.

#### ARTICLE 12. LOADS ON ROOF TRUSSES.

The load which a roof truss is called upon to support is made up, in part, of the weight of the truss itself, and, in part, of the roofing materials such as wood, slate, tin, copper, tile, corrugated or flat steel, gravel-tar or other kinds of patented roofings.



The load per square foot of such roofing materials being known, and the distance between the trusses being assumed, it will be a comparatively easy matter to determine how much load must be carried by each truss and, in turn, how much load must be carried by each joint of that truss. It



**Fig. 11.**

Shaded portion of the roof is being supported by a single roof truss. (A) Knee-brace. (B) Portal type of lateral bracing. (C) Diagonal lateral bracing in the side walls. (E) Lateral bracing in the plane of the roof. (F) Diagonal bracing in the plane of the ceiling or lower chords.

is readily seen that one of the interior trusses, and not the end truss of a building, is the one to be considered as one of the series of trusses which support the roof. See Fig. 11.

Other kinds of loads on roof trusses, such as wind loads, snowloads, and special loads are discussed in some of the following articles.

With the total steady load known, it will be observed that the proper distribution is made by dividing the total load by the number of members exposed to load, and then placing this fraction at each of the joints, excepting the end-wall joints, where only one half as much load acts. The above statement is true when the load is uniformly distributed and when the members exposed to load are of equal length.

In general, it might be pointed out that, the load on any member is divided equally between the two adjacent joints etc.

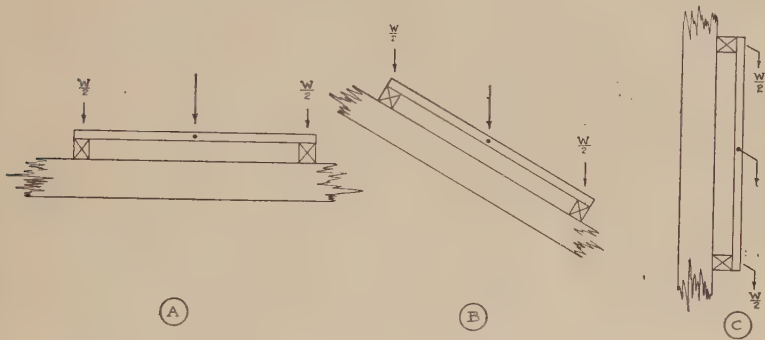


Fig. 12.

The beam or sheathing *A* Fig. 12, whose weight is equal to  $W$ , is supported on two purlins, each supporting one half of the total load  $W$ . It makes no difference whether the beam is horizontal, inclined, or vertical so long as it is fastened to the purlins. *B* and *C* of Figure 12 illustrate these cases.

In Figure 13 *A*, *B*, and *C* there are shown three possible conditions of loading that may exist on a roof, and the method of arriving at the proper distribution of those loads is as follows:—

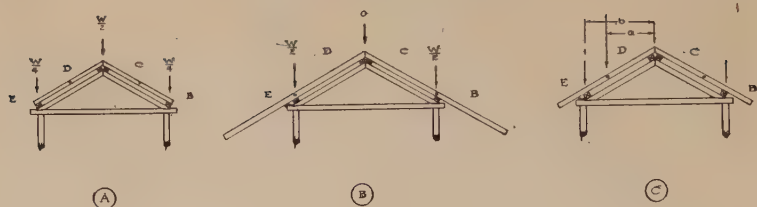


Fig. 13.

Case (A)  $BC = \frac{W}{4}$ ,  $CD = \frac{W}{2}$  and  $DE = \frac{W}{4}$

Case (B) When the tail of the rafter is equal in length to the rafter proper the loading is thus:—

$$BC = \frac{W}{2}, \quad DE = \frac{W}{2} \text{ and } CD = 0.$$

The center of gravity of the load being above the point of support and the rafter being rigid.

Case (C)  $BC$ ,  $CD$ , and  $DE$  each carry part of the load, depending on the relative lengths of the rafter and the tail. The simplest way in which to determine the value of the loads  $BC$ ,  $CD$  and  $DE$  is by means of the method of moments as outlined in in article 16.

Taking moments about the apex of the roof as a center, we have:—

$$\frac{W}{2} \times a = DE \times b \quad \text{or} \quad DE = \frac{Wa}{2b} = CB$$

$$CD = W - (DE + CB)$$

While there are tables in many of the handbooks, giving the weight per foot of the different types of roof truss for various spans, it will be more satisfactory to arrive at

the approximate weight of any truss or structure by consulting tables of weights of materials of standard size and shape and from these calculate the weight of the truss. See CAMBRIA.

WEIGHTS PER SQUARE FOOT OF ROOFING MATERIALS.

Following, are the approximate weights of the more common kinds of roofing materials. Exact weights for any particular type of roofing can usually be learned by personal investigation or by consulting trade catalogs, or it may be specified by the designer.

| Material                             | Weight per sq. foot |
|--------------------------------------|---------------------|
| Corrugated Black Iron .....          | 0.9 — 3.28          |
| Corrugated Galv. Iron .....          | 1.21— 3.88          |
| Pine shingles (4½" to weather) ....  | 1.44— 2.16          |
| Skylight glass .....                 | 2.5 — 7.00          |
| Slate ¼" thick lapped .....          | 12.0 —14.00         |
| Tile ⅝" lapped .....                 | 14.0 —10.00         |
| Tin .....                            | .8 — 1.25           |
| Felt and gravel .....                | 8.0 —10.00          |
| Lath and plaster .....               | 9.0 —10.00          |
| Yellow pine sheathing 1" thick ..... | 3.0 — 3.5           |
| Sheet lead .....                     | 5.0 — 8.00          |
| Zinc .....                           | 1.0 — 2.0           |

WEIGHT OF SNOW ON ROOFS.

The weight of snow which any roof will be called on to support is of a rather indefinite quantity. It depends primarily upon the amount of snowfall in the locality, the shape of the roof, the location of the roof with respect to

objects which might tend to increase or decrease the deposition of snow on the roof, whether the building is being heated enough to melt the snow as it falls, etc.

There is considerable difference of opinion in regard to the quantity of snow to be allowed for when designing a roof. With these opinions in mind, it seems reasonable to expect that under normal conditions in the Lake Superior region, from 10 to 30 pounds of snow per square foot of horizontal projection of roof.

Usually we assume that snow will not stay on a slope higher than  $60^{\circ}$ , but there are times when considerable snow or sleet will adhere to a vertical surface.

In the design of any building where snow or ice load may be expected, much advantage may be gained through the knowledge of the prevailing winds, and of the cross and eddy currents of air. It very often happens that we will find a nearly flat roof practically free from snow, while nearby there will be a pitched roof with one side swept clear by the wind, but with a heavy packed coating of snow and ice on the other side. This coating has been observed to be as much as two feet thick and packed so that it weighed 30 pounds per square foot of horizontal projection of the roof.

A roof that is continually cold, or one covering an unheated building, will be readily swept clean of snow by the wind, provided the shape of the roof will permit.

When driving snow strikes a moderately heated building, it is apt to adhere and accumulate very rapidly, and when the outside temperature rises, will melt and refreeze on the eaves of the building, making ice and icicles which back up the water and cause the roof to leak. The above named difficulties may usually be overcome by keeping the eaves



warm, or better still, by providing inside drainage so that the water from the melting ice and snow will be conducted to the sewer before it can refreeze.

#### ARTICLE 13. EXAMPLE SOLUTIONS OF A SIMPLE TRUSS.

##### Problem 10.

Problem 10 represents the simplest form of triangular truss used for small spans where no difficulty is likely to be encountered from the sagging of the inclined members or rafters. When solving the problem, the student should note carefully the effect of changing the angle of the reactions, not only upon the reactions themselves, but upon every member of the truss.

##### INCLINED REACTIONS.

Suppose that the roof has a certain load per running foot of rafter, and let the whole load be denoted by  $W$ . It is evident that one-half of the load on the rafter  $CF$  will be supported by the joint  $B$ , and one-half by the upper joint. The same will be true for the rafter  $DF$ . Therefore the joint  $B$  will carry  $\frac{1}{4}W$ , the upper joint  $\frac{1}{2}W$ , and the joint at  $E$ ,  $\frac{1}{4}W$ . The additional stress produced in  $CF$  by the bending action of the load which it carries, is not considered at this time, but may be noted and allowed for separately.

Taking the external forces in order from left to right over the roof, lay off  $bc$ , or  $\frac{1}{4}W$ , vertically, to represent the weight  $BC$  acting downward at the joint  $B$ ; next  $cd$  equal to  $\frac{1}{2}W$ , for the weight at  $E$ . Call  $be$  the load line. Let the two reactions or supporting forces, for the present, be considered as inclined  $10^\circ$  from the vertical, as shown in

Prob. 10. Since the truss is symmetrical and is symmetrically loaded, the resultant of the load must pass through the apex of the roof, and, since the two supporting forces must meet the resultant at one point, the two reactions must be equal in value and equally inclined.

To complete the polygon of external forces we continue reading about the frame in a counterclockwise direction and draw the line  $ea$  up from the extremity  $e$  of the load line and parallel to the upward reaction  $EA$ ; and lastly, a line  $ab$  parallel to the other reaction  $AB$  to close on  $b$ , the point of beginning.

#### STRESSES IN THE FRAME.

In this truss we find the stresses by beginning at any point; let us begin at  $B$ . Here we have equilibrium under the action of four forces, of which the two external ones are known. Taking the latter in the same order as above, and beginning at  $b$ , pass over  $bc$  downward; then draw  $cf$  parallel to  $CF$ , in such direction that  $fa$ , drawn from  $f$  parallel to  $FA$ , will strike the point  $a$ . Then the line  $ab$  will be parallel to  $AB$  and act as a closing line for the polygon, (The point  $a$  might have been previously located by knowing that  $AB = MA$ .)

Considering the joint at the apex of the roof, and again taking the forces in the same order, pass down along the line  $cd$  for the external force, then up to  $f$  for the thrust  $df$ , and finally draw  $fc$  parallel to  $FC$ , thus determining the thrust of that rafter against the top joint. If the line does not close on  $c$ , the drawing has not been made with care. As all of the stresses have now been determined, we need not examine the remaining joint. It may again be noted that we pass over the stress lines in one direction when we

analyze the stresses at the joint at the end of the member to which the line is parallel, and in the reverse direction when we consider the joint at the other end of the same member.

#### INCLINED REACTIONS.

If the supporting forces had been more inclined from the vertical, the point of intersection of the reactions  $a$ , would have been nearer to  $f$ , thus diminishing the tension  $AF$ , but not affecting the compression in the rafters. The inclination might have been so much increased that  $a$  would fall on  $f$ , when the member  $AF$  would have no stress, the thrust of the rafters being balanced without it. If  $a$  fell to the right of  $f$ ,  $af$  would be in compression instead of tension. If the angle of the reactions from the vertical is zero, the value of  $EA$  and  $AB$  will each equal  $\frac{1}{2} W$  and the point  $a$  will be in the middle of the load line, thus closing up the polygon of external forces in such manner that it becomes a straight line which is divided into fractional parts of the load  $W$  and the reactions.

When the distance to be spanned is from 20 to 40 feet, the KING POST TRUSS Prob. 11, Fig. 14 will be found to be the simplest and most common type of truss to construct of either wood or steel. The roof load may be concentrated on large purlins placed at the joints in the rafter, but quite often it is distributed at shorter intervals by smaller purlins so that the principal rafter is subjected to cross-bending loads as well as the transverse stress along its axis.

The total load on the truss may be computed by knowing the value of  $\beta$  and the load per running foot of rafter, and it is equal to:—

$$W = \frac{\text{Span}}{\cos \beta} \times \text{load per running foot of rafter.}$$

The amount of load to be placed at each joint may then be found by dividing  $W$  by 4 (the number of members subjected to load) and placing  $\frac{1}{4} W$  at each of the joints excepting the two end joints, which carry only  $\frac{1}{8} W$  each.

The truss, being symmetrical and having a symmetrical load on it, will be supported by the two end reactions, each

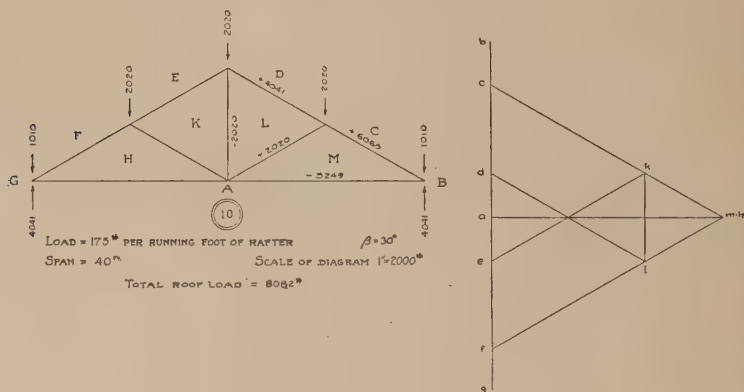


Fig. 14.

of which is equal to  $\frac{1}{2} W$ . Lay off the polygon of external forces (load line) by beginning at  $B$ , and to a scale that will result in a final stress diagram, which is somewhat larger than the frame diagram; lay off the loads in the order  $bc$ ,  $cd$ ,  $de$  and  $fg$ , each downward, for that is the direction in which the loads act; then draw the reactions  $ga$  and  $ab$  upward to close the load line and to check the sum of the several measurements. Then, by analyzing one joint at a time, the several triangles or force polygons for each joint may be drawn, using some of the previously determined values as a base for the determination of the unknowns.

At the right hand end of the truss we already have the values of the load  $BC$  and of the reaction  $AB$  represented on the stress diagram, and continuing to follow in a counterclockwise direction, we draw  $cm$  parallel to  $CM$  and  $ma$  parallel to  $MA$ , and in this way find the location of the point



Fig. 15.

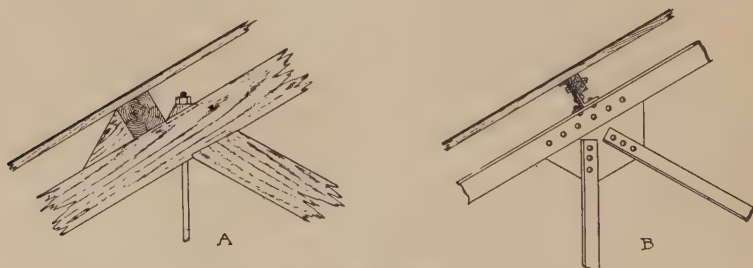
Simple truss of steel used to support slab roof. Note the joints, Struts, Ties, Principal Rafters and Purlin. Note also the method of supporting traveling crane track on the sides of the building by means of inside pilasters. Span of truss about 50 feet.

$m$ , which fixes the magnitude of the stress in the members  $CM$  and  $MA$ .

It will be well to state here, that, in order to avoid becoming confused, one should NOTE, before attacking any



joint, whether or not more than two unknown values exist at that joint. If there are more than two of such, then the solution of that joint must not be attempted. Also, before going on to the second joint of the series, be sure that you have read the designation of the members in the proper order about the joint. (Say in a counterclockwise direction.) The solution of the remaining joints of the truss is made in the same way as for the joint *ABCM*. Continue in this manner until all of the joints have been solved and until a check line has been reached to show you that the stress diagram is consistent with the frame diagram. When the problem has been completed, it should have all data given as illustrated in Fig. 14.



**Fig. 16.**

Comparing methods of supporting the purlins on wood and steel roof trusses.

#### **SUPERFLUOUS MEMBERS.**

In making an analysis of problem 15, it will be found that there is no stress in the members *YZ* and *NO*. At each of these lower joints we have a condition of equilibrium in which three forces appear to participate, but since two of these forces are parallel and act along the same line, the third can have no value. Such members are generally designated as Superfluous Members.

There are many trusses which, when analyzed, will show that they are made up in part of this type of member and, although theoretically they have no stress, the subsequent steps in the design of a structure will show how they may be used to good advantage in providing support for the longer members to prevent their sagging or buckling. See problems 15, 16 and 21.

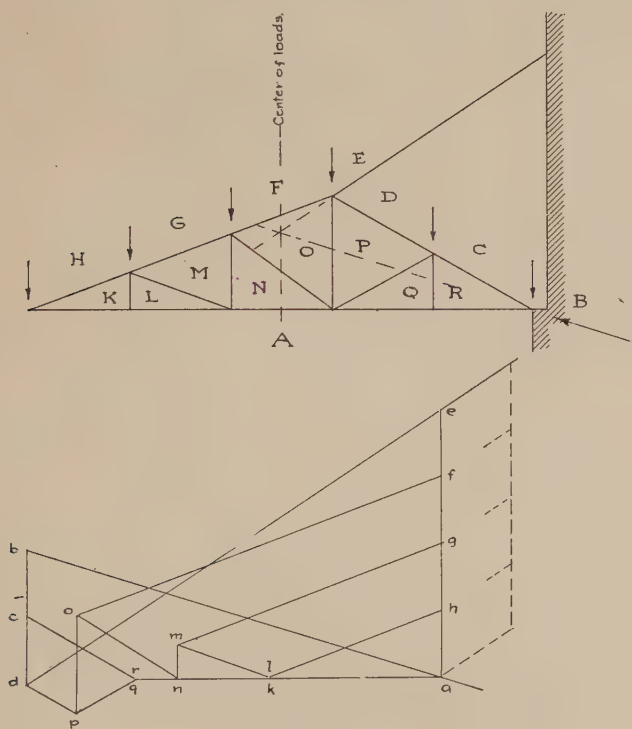


Fig. 17.

In the problem illustrated by Figure 17, we have something out of the ordinary, inasmuch, as while the loads are vertical and are uniformly distributed, the reactions or wall supports are both on the same side of the truss so that the roof is really supported from the one vertical wall

which must be rigid enough to furnish support or it must, in turn, be supported by other forms of trussing. This latter is the usual case. This type of structure is often used to support the roofing over an area where posts and columns would be objectionable as for instance:—on loading or transfer depots where cars and trucks and their loads of merchandise must be protected from the weather.

There is one group of vertical forces and two other external forces acting on the truss. So far as equilibrium of the truss is concerned, the group of vertical loads may be considered as one whose magnitude is equal to the sum of these loads in that direction and acting as a resultant of those loads at their center of gravity. In the case here illustrated, the loading is uniform and symmetrical about a vertical line drawn through the center of the span. This resultant line, representing the vertical forces intersects the second external force  $EF$ , and with that point of intersection fixed it follows that the third external force acting at  $B$  must meet the other two a common point, thus fixing the line of action of that third external force.

The load line for this set of conditions will be one which will represent graphically the forces acting upon the frame and which are in equilibrium. In problem 11 we had numerical values given for all of the external forces acting on the frame but here we have two external forces whose values are unknown.

Begin to lay off the load line by drawing  $bc$ ,  $cd$  and  $de$  to scale and then draw  $ef$  parallel to  $EF$ . Since we do not know the exact location of the point  $f$  in the stress diagram, suppose that we choose one at  $f^1$  and then proceed to lay off the forces  $fg$ ,  $gh$ ,  $ha$  and  $ab$ . Since we choose one point in the polygon at random, we could not expect the load line to

close, but it immediately becomes evident that we can modify our choice, as indicated by the dotted line, in order to show equilibrium in the stress diagram and at the same time be consistent with the frame diagram.

With the load line closed, the remainder of the solution is quite simple.

In case the loading on the roof is not uniform or not symmetrical, it would be necessary to find the center of gravity of the group of parallel forces by the method of moments as outlined in article 16.

## QUESTIONS AND PROBLEMS.

1. What change may take place in the member  $FA$  of problem 10 and of what practical value is this?  
How is the rafter stress affected?
2. Why is the truss in problem 11 called the KING POST TRUSS?  
Is  $KL$  + or —?
3. What are the values of the members  $HL$  and  $LM$  in problem 12?  
Explain why, and state the conditions under which it is not true.
4. What is the value of the stress  $NO$  in problem 15? Why is this member placed in the frame?
5. In what kind of a structure may the frame of problem 16 be used? What is the first essential point to be observed when making the graphical analysis of this problem?
6. Compute the total load and the reactions for a roof truss like problem 15 when the load per running foot of rafter is 15 pounds,  $\beta = 24^\circ$  and the span 32 feet.

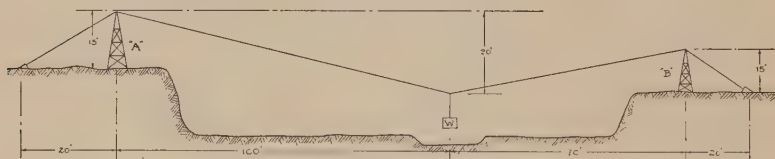


Fig. 18.

7. a. In the above figure, if  $W$  equals 2000 pounds, how would you proceed to find the vertical stress on the towers  $A$  and  $B$ ?
- b. How would you determine the horizontal thrust on the tops of the towers  $A$  and  $B$ ?
- c. How would you determine the stress in the anchor or guy lines?
- d. If the tensile strength of the rope is limited, how would you proceed to determine the size of the load that could be lifted?
- e. How could the arrangement be modified so that it would be possible to lift greater loads without increasing the tension in the rope? Demonstrate all graphically.
8. A ladder weighing 80 pounds is 30 feet long and stands leaning against a smooth brick wall so that its base is 12 feet from the wall. When a 150 pound man climbs the ladder and is within five feet of the top, what horizontal thrust will be required to prevent the base of the ladder from slipping?



## CHAPTER IV.

### ARTICLE 14. SOLUTION BY SYMMETRY.

The FINK truss, Problem 17 and 18, is best adapted to steel construction and is frequently constructed with the lower chord of the truss cambered (raised at the center), as shown in the left hand portion of the frame in figure 19 and by problem 18.

Cambering the lower chord of the truss in this manner will give more headroom and a somewhat better looking truss, but at the expense of greater stresses in certain members of the truss. These changes in stress may best be observed by careful comparison of the stress diagrams of the cambered and uncambered trusses when all of the other conditions are the same.

Trusses of this type are common in buildings where the span is from 40 to 75 feet, and are, in some cases, placed under more than the ordinary roof loads, in that, in addition to providing shelter from weather, the truss will be called upon to support loads such as suspended traveling cranes, trolleys, line shafting etc. These loads, when they exist, are placed at the joints of the lower tie whenever possible. See prob. 20. When only the roof loads prevail, the total load is found by knowing the span, the angle of the rafter or pitch of the roof, the distance between trusses and weight of a square foot of roofing. See Fig. 11. The loads are distributed uniformly so that  $\frac{1}{8} W$  is placed at

each of the rafter joints excepting the two end joints where  $\frac{1}{16} W$  acts. The reactions  $R_1$  and  $R_2$  are each equal to  $\frac{1}{2} W$ .

Lay off the load line complete as shown in Fig. 19 and solve joint  $BCZA$ , then the joint  $CDYZ$ , then  $AZYX$ , after

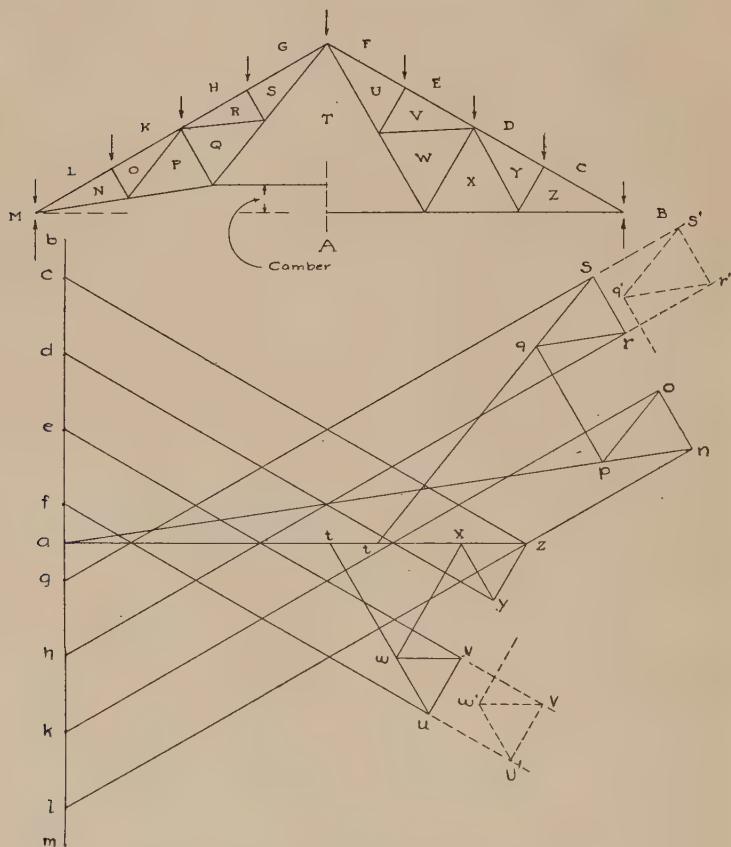


Fig. 19.

The FINK truss with stress diagram.

which we find that the next joints appear insoluble, since there are three unknown values to be determined in each case.

We will now resort to a special method of solution which depends upon the symmetry of the frame, and attempt to apply the ratio of size and direction of the members of a frame to the size and direction of the stresses shown by the stress diagram.

The right hand half of the frame is symmetrical about the line  $WX$  and the loading  $DC = EF$ , and since  $YZ$  is placed under conditions similar to those of  $UV$ , we could expect the stresses  $yz$  and  $uv$  to be the same. Draw the line  $ev'$  parallel to  $EV$ , and  $fu'$  parallel to  $FU$ . Then, by inspection of the stress diagram, it is evident that  $v'u'$  will be the same value no matter how the value  $ev$  may be varied, and also that  $vu'$  is equal to  $yz$ , since they are the intercepts between pairs of lines that are parallel and equidistant, for the load  $cd$  equals the load  $ef$ .

Similarly, the stress in  $VW$  may be expected to be found equal in value to that of  $XY$  and of the same sign (tension). Then, in order to satisfy the fixed conditions, such as the location of  $x$  and of the direction of  $wx$ , it will be necessary to shift  $w'$  to  $w$  and  $v'$  to  $v$ , and here symmetry once more is in evidence, since  $zy$  and  $uv$  are in line so that the values of the stresses  $cz$ ,  $dy$ ,  $ev$  and  $fu$  decrease in regular steps.

A check on the diagram may be made by completing the stress diagram for the entire frame, using the value of  $TA$  as an aid in solving the left hand part of the frame by the usual methods.

The left hand half of the frame of figure 19 is intended to show the changes that take place in the stresses of the frame when the truss is cambered, while the comparison between the upper and lower halves of the stress diagram will indicate the changes in stress due to the cambering of the truss.

## ARTICLE 15. TRUSSES WITH INTERIOR LOADING.

A few of the conditions applicable to the truss of problem 20 are indicated on figures 21 and 22. In many shop buildings it is desirable to be able to support some of the interior loads from the roof truss instead of providing



**Fig. 20.**

Typical Fink Truss supporting roof over a "Dry" or Changehouse. Note the structural details of the different members of the truss and also the lateral bracing in the plane of the lower ties of the trusses. Span about 40 feet.

separate means of support for each of the units. Figure 22 shows how the line shafting in a machine shop may be supported, and how the stresses in the belts which lead to the various machines may effect the truss. There may be

several of these line shafts or counter shafts suspended from the truss, and in each case the weight of the shaft, pulleys and bearings, as well as the tension in the belt, must be given consideration.

On the other half of the figure is shown a traveling trolley, which travels on the lower flange of an "I" beam



**Fig. 21.**

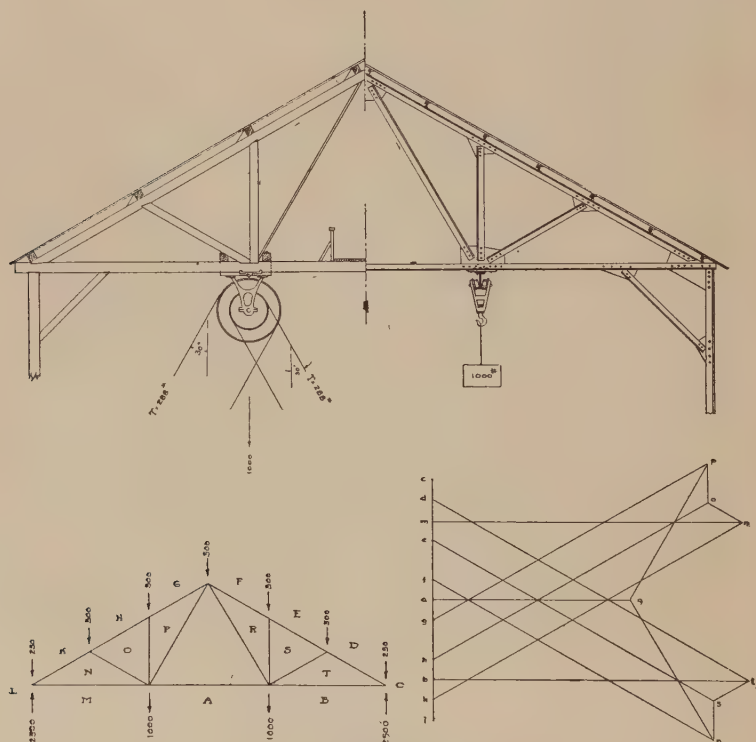
Heavy type FINK truss for blacksmith and machine shop, arranged to carry traveling crane to serve the middle floor. Suspended Jib crane at right is for blacksmith service; Note that it swings in a circle and that a chain-block is suspended from the traveler or trolley.

fastened to the trusses. This trolley will carry a moving load, and is often used to lift parts of machinery from a wagon or truck on the floor of the shop or at the door and to convey such parts to the drill-press, lathe, shaper, forge or other machine tool used for working on the part in question.



In addition to these two common types of loads we may have others, such as suspended motors, hand-trolleys, forge hoods etc.

While many loads of this type are small in comparison with the roof load and do not require much additional



**Fig. 22.**

strength in the roof truss, it is well to give them all due consideration when making the design of a building or truss.

The manner of treating the suspended loads and the relative effect of these loads on the stresses of the frame may



best be learned by a careful study of the accompanying stress diagram. Note that the load line at first does not appear to fully represent the total loads, but investigate and find the reason for this.

#### ARTICLE 16. THE THEORY OF MOMENTS.

It often happens that the roof truss or other structure is either unsymmetrical or has an unsymmetrical load, in which case the most convenient method of finding the numerical value of the two supporting forces (reactions) is by means of moments. The meaning and manner of using the method is briefly stated and illustrated, as follows:—

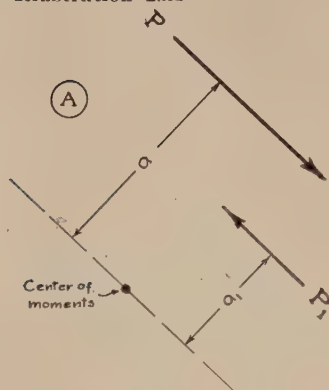
Conditions:—

- (a) When the forces acting on any frame are in equilibrium, the resultant of all is zero.
- (b) The sum of the vertical components is also equal to zero.
- (c) The sum of the horizontal components is also equal to zero.

Definitions of terms:—

- (a) The moment arm of any force is the perpendicular distance from the line of action of that force to the center of moments.
- (b) The moment of a force is the product of the force and its moment arm.
- (c) The sum of the moments about a point must equal zero when the forces are in equilibrium.

Illustration 23A

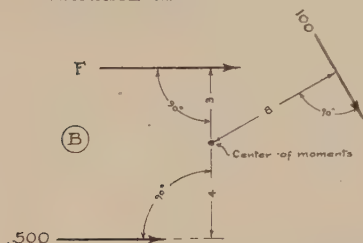


$$+ (P \times a - (P_1 \times a_1)) = 0.$$

$$\text{or } Pa = P_1 a_1.$$

$$P = \frac{P_1 a_1}{a}$$

Illustration 23B



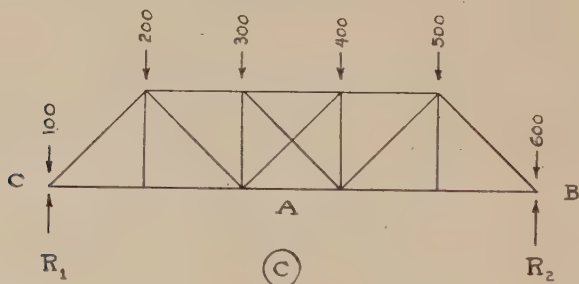
$$+ (100 \times 8) + (F \times 3) - (500 \times 4) = 0.$$

$$800 + 3F - 2000 = 0.$$

$$3F = 1200.$$

$$F = 400.$$

Illustration 23C



When the span line is divided into equal parts, we get, by taking moments about the left hand support, the equation:—

$$(100 \times 0) + (200 \times 1) + (300 \times 2) + (400 \times 3) + (500 \times 4) + (600 \times 5) - (R_2 \times 5) = 0.$$

Solving for  $R_2$ , we get,

$$R_2 = 1400.$$

And about the right hand support:—

$$\begin{aligned} &-(600 \times 0) - (500 \times 1) - (400 \times 2) - (300 \times 3) - \\ &\quad (200 \times 4) - (100 \times 5) + (R_1 \times 5) = 0. \end{aligned}$$

Solving for  $R_1$ , we get,

$$R_1 = 700.$$

Check:—

$$\begin{aligned} \text{Total load} &= (100 + 200 + 300 + 400 + 500 + 600) \\ &= R_1 + R_2. \end{aligned}$$

$$2100 = 1400 + 700 = 2100.$$

#### ARTICLE 17. TRAPEZOIDAL SHAPED AND BRIDGE TRUSSES.

The loads on the different joints of the truss, Problem 21, are apportioned according to the lengths of the several members, which are subjected to roof load so that  $FE$  bears  $\frac{1}{2}$  the load on the member  $EG$ , while  $ED$  is the load made up of the sum of  $\frac{1}{2}$  the load on  $EG$  and  $\frac{1}{2}$  the load on  $DK$  etc.

Where, as in cases 2 and 3, the extra loads occur on the joint  $CD$ , the reactions are increased, but not equally, since the said extra load is not symmetrically placed. See figures 9 and 24.

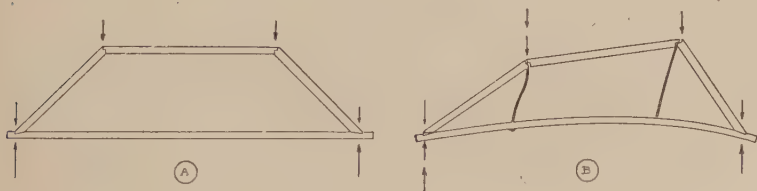


Fig. 24.

Illustrating manner of failure of a trapezoidal shaped truss with no interior bracing, when unsymmetrically loaded and when the rigidity is dependent upon the stiffness of certain members of the truss.

The true values of the reactions due to the extra load may be taken to be inversely proportional to the distance of the load from that point of support. A better way is to compute the values of the reactions by the method of moments outlined in article 16.

The three cases are given so that the student will have an opportunity to study the effect of reversing the diagonal on the various members of the frame. The information



**Fig. 25.**

Typical Pratt truss for highway bridge. Note the truss members and lateral bracing in the plane of the upper chords, and the portals between opposite posts and also between the End top chords. Also note the concrete Abutments and wings to prevent erosion of the approaches during high water.

gained through their solution will prove useful in the subsequent analyses.

The PRATT type of truss Fig. 25 is one of the most common in present day use for bridge and roof building of moderate span. When used in bridge construction, two such trusses are placed side by side and held securely in position

by means of cross griders and "Portals". See prob. 41. The "Deck" load in this case is carried on the griders just above the plane of the lower chord of the truss. See Fig. 29.

In the PRATT truss the diagonals in the panels are in tension, while in the HOWE truss Problem 23, they are in compression. On careful consideration of these two trusses and their stress diagrams it will be seen that the HOWE type is better adapted for composite timber construction, since the tension members (tie rods) will be vertical and the compression members inclined or diagonal, which combination makes it easy to construct the joints. See Fig. 26. The tension members in composite construction are

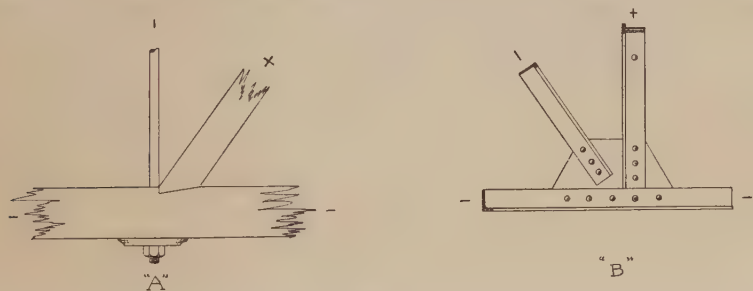


Fig. 26.

Typical joints in the lower tie beam of a roof truss. The "Pratt" type "B" is generally built of steel, while the "Howe" type "A" is built of wood.

generally round steel rods or bolts, which extend through a hole in the lower timber chord and have suitable metal washers under the head of bolt and under the nut.

In the PRATT truss the vertical members are in compression and the diagonals in tension. This condition makes this type of truss adaptable to steel construction since it is a comparatively easy matter to provide long tension members and to make the necessary joints.

The WARREN GIRDER or TRUSS. Problem 24 is one which may be used in the construction of all kinds of flat roofs, and for bridge construction, particularly for highway bridges of moderate span.

In its simplest form it consists of a series of equilateral triangular panels, the size of which depends largely upon



FIG. 27.

Modified Warren Girder, a type commonly used in the construction of highway bridges of moderate span.

the span and on the load to be supported. When the sides of the triangular panels increase beyond the length of about fourteen feet, it is common practice to use intermediate vertical members extending up from the lower chord joints or down from the upper chord joints, or both. In the first case, the effect is to add members which are



theoretically superfluous but which really support the upper chord members and make them more rigid, while in the second case the extra members support the lower chord members and shorten the span of the lateral girders which carry the deck load.

Figure 9 will serve to show the manner in which the Warren Girder may be used in the construction of a wooden roof, while Figure 27 illustrates its use as a highway bridge.

After drawing the stress diagram of this truss with both the roof and ceiling loads, it is well to make comparisons with the stress diagrams of the Howe and Pratt trusses.

The BOW-STRING TRUSS, Problem 25 is another example of a popular type of bridge used on highways, the load being carried on the deck just above the lower chord and between a pair of such trusses.



**Fig. 28.**

Typical Bowstring truss in the foreground and a Plate-girder truss or bridge in the background.



**Fig. 29.**

Detail of Bowstring truss. Note such details as Upper-chord, Lower-chord or Eye-bar, Diagonal Eyebar, Deck beam, Floor joist, Diagonal Deck-bracing and Piers.

The PARABOLIC BOW-STRING TRUSS while theoretically, or rather statically efficient, is not well suited to actual construction since the additional cost for providing suitable suspension of the deck is high.

The comparison of the stress diagram of these two trusses will illustrate the effect of increasing the depth of the truss.

Figure 28 illustrates the bowstring type truss as used on a highway bridge.

The "K" Truss as illustrated by figure 30 is one which has been used in recent years in the construction of bridges of great span. The system of having two diagonals extending from the middle of one post to the top and bottom of the next post was used in the design of the well known Quebec bridge. The "K" type construction is not economical for spans of less than three hundred feet, but in such long spans the panels become so high that considerable advantage is gained since the members are shorter than would be the case if the Howe or the Pratt type were used.

The frame, Figure 30 is made with one-half "K" type design while the other half has the Pratt design, so that by comparing the two halves of this frame we may observe the relative lengths of the members in similarly placed panels, and by referring to the stress diagram of the same figure, observe the stresses induced in the two types of construction under like conditions. In all of these bridge trusses, due consideration must be given to the fact that the loads carried by them are moving. The amount and manner of loading which is generally used by designers of railroad bridges has been standardized.

While the construction of such long spans as mentioned above is rather unusual, the example will serve to suggest

how sizes of panels and lengths of members of any system of trussing may be modified in order to make them shorter and more economical to build. This point may well be taken into consideration when the engineer is making a design for a head frame or similarly shaped structure.

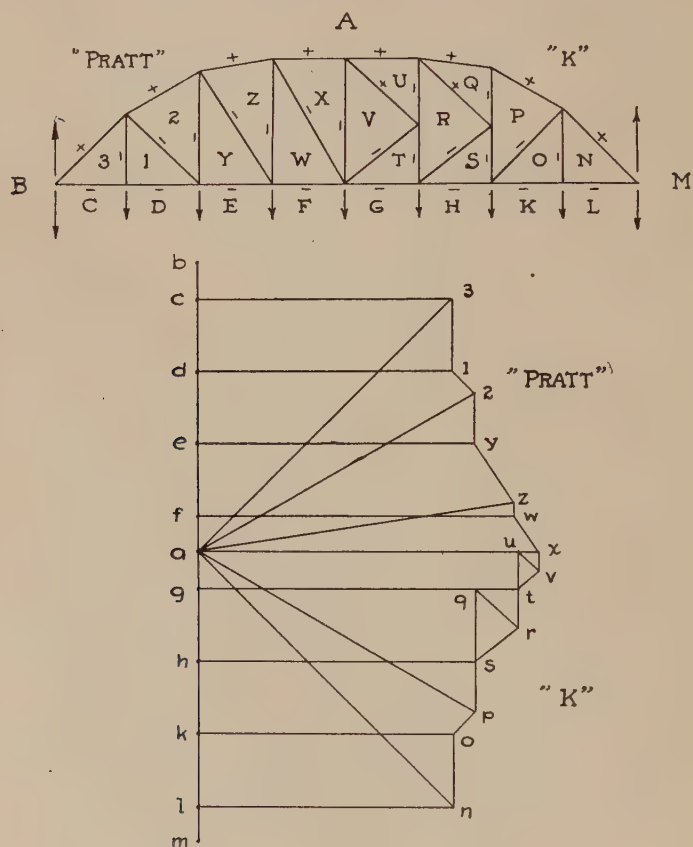


Fig. 30.

## ARTICLE 18. THE SWING OR DRAWBRIDGE.

A SWING or DRAWBRIDGE is a movable structure placed at the intersection of two lines of traffic, where it is necessary to move that structure quickly at the will of the operator to such position as will serve either of the two lines of traffic. It consists of one or two spans, pivoted so that they may swing readily from the position in a bridge to one at right angles to the bridge, thus allowing boats to pass through the opening. It is not designed to carry any of the usual live or moving loads when it is swung open, but when closed, its span is really shortened so that instead of a single truss supported on the central pier, we will have two spans supported on the central and two adjacent piers, thereby greatly increasing the carrying capacity.

The machinery for operating the swing bridge, its gates, locks, and other accessories is often located in a housing above the central pier, where the operator may have it in sight and at the same time get a view along the lines of traffic that may make use of the bridge. The span is generally arranged so that when it is about to swing open, all deck traffic is stopped and the end supports withdrawn or the struts at the ends of the span lifted, so that the two halves of the swing bridge balance each other and derive all of their support from the central pier. When swung open to accomodate navigation, they are usually protected from injury or danger of being overthrown through collision by being swung above a pier several feet larger than the swing bridge and better able to withstand shocks from colliding boats and vessels.

The different conditions for loading this bridge and the resulting stresses are illustrated in figure 31.

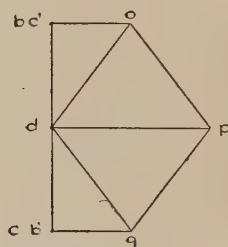
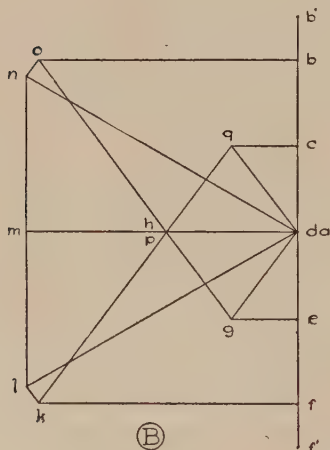
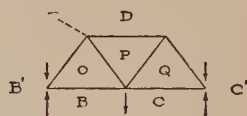
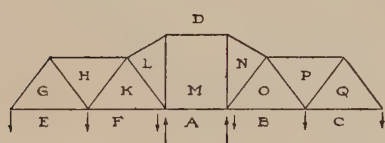
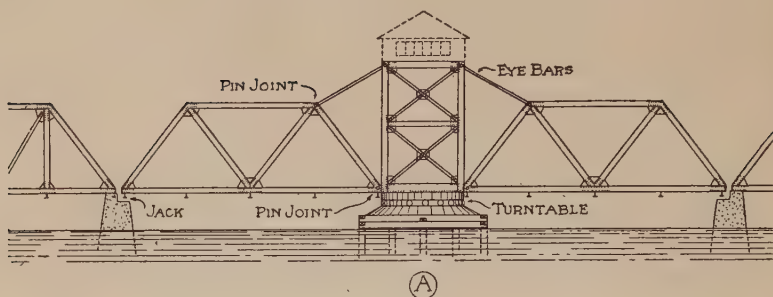


Fig. 31.

## ARTICLE 19. THE SCISSOR TRUSS.

The scissor truss, Problem 28, represents a type of construction that may be applied to trusses having a span from 25 to 35 feet. There are a few interesting points to be observed when making the determination of the stresses



existing in this kind of truss and these may best be learned by close observation of the stress diagram.

If we proceed with the distribution of the steady roof load and wall reaction in the usual way, there is no apparent difficulty until we get a diagram as shown in *C* Figure 32, where there are two locations for the point *KL*, indicating that there must be something wrong with the figure or with the method.

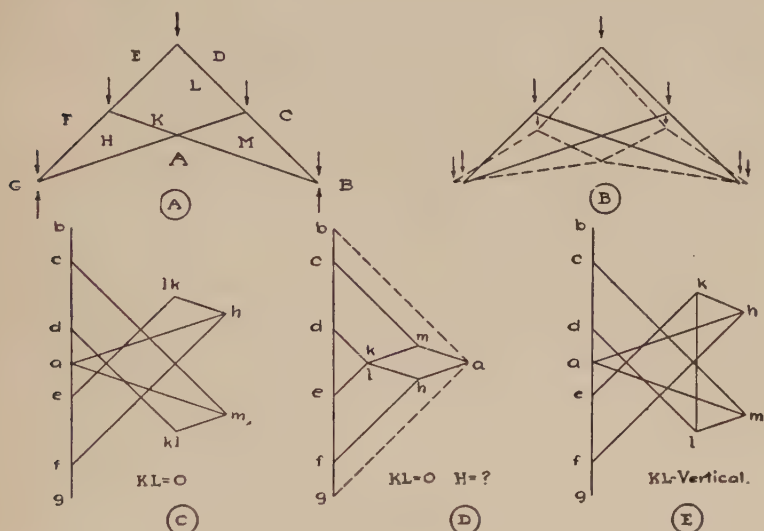


Fig. 32.

On careful inspection of the truss we find that the upper panel *KL* has four sides, pin joints, and therefore, when we undertake to draw the stress diagram, we are basing our action on an assumption that it is a stable truss, which, of course, is not true on account of the four sided panel with the pin joints.

In order to make this truss stable and capable of supporting roof loads it will be necessary to provide some means of preventing the wall joints *C*, Figure 32, from sliding

horizontally. In other words, the walls of the building must supply the necessary horizontal thrust, or, the reactions must have a certain inclination so that the summation of the forces in any or all directions shall equal zero.

Taking it for granted that this requirement may be fulfilled, we may set about determining the amount of thrust by assuming that there is just enough thrust to prevent sliding at the wall joints, and solve the joints *DEKL*; then *EFHKL*; then *CDLKM* and finally *LKHAM*, which will give us the location of the point *a* on the stress diagram. The reactions *FA* and *AB* may then be resolved into vertical components whose values are each equal to one half the total load, and the horizontal components which represent the amount of horizontal thrust that the walls will be called upon to produce.

In order to avoid the necessity for horizontal thrust or the bending strain in the members, a diagonal member *KL* may be placed in the four-sided panel in either a horizontal or vertical position. By the addition of this member *KL*, the conditions are greatly changed and the need for horizontal wall thrust will be eliminated.

The stress diagrams for all of the cases of this problem should be constructed to the same scale and located on the sheet in such manner that the student may readily see what variations take place in the different diagrams and be able to learn the cause therefore.

The knowledge gained from these observations will be found valuable when making the solutions for different types of Scissor trusses.

## ARTICLE 20. THE TWO-HINGED ARCH, PROB. 29.

The truss, as shown on the problem sheets, has nothing to indicate that it might be different from the ordinary, for it is capable of supporting steady roof loads even though they are not symmetrically placed. Ordinarily, the reactions for the support of the truss would be vertical, but if a horizontal thrust can be supplied by the supporting walls it will tend to decrease, materially, the stresses in the frame. When this thrust exists, the truss is sometimes designated as a Two-hinged arch.

In drawing the stress diagram we learn the value of the stress in the member  $AL$ . Should we now wish to reduce the value of the stress in this member  $AL$ , we may do so by moving the point  $a$  of the load line to the right and thus make the reaction  $FA$  and  $AB$  inclined instead of vertical. The distance that  $a$  is moved, or rather, the amount of horizontal thrust supplied by the wall, may be varied to give a still smaller stress in  $AL$  or even to change the sign of the stress.

The horizontal thrust in this case may be likened to a lower tie or chord, which changes the "depth" of the truss and thus decreases the stresses in the frame. See effect of camber on problem 18.

## ARTICLE 21. THE THREE-HINGED ARCH.

The three-hinged arch consists of two arched ribs hinged at the crown as well as at the two abutments. See problem 30. The outward thrust at the ends may be resisted by the abutments or by a tie-rod joining the two ends of the truss. The ribs may be solid, but in most cases will be built up of a series of rigid triangular panels.

This type of truss is a good one for buildings of great span as well as for bridges. It is a type which evidently cannot stand, even under dead vertical loads, without the aid of horizontal thrust applied at the abutments. In this respect it is like the arch, and on account of the hinges, it illustrates the only statically determinate type of arch construction.

The distribution of the dead or roof load and of the weight of truss offers nothing new, but the determination of the

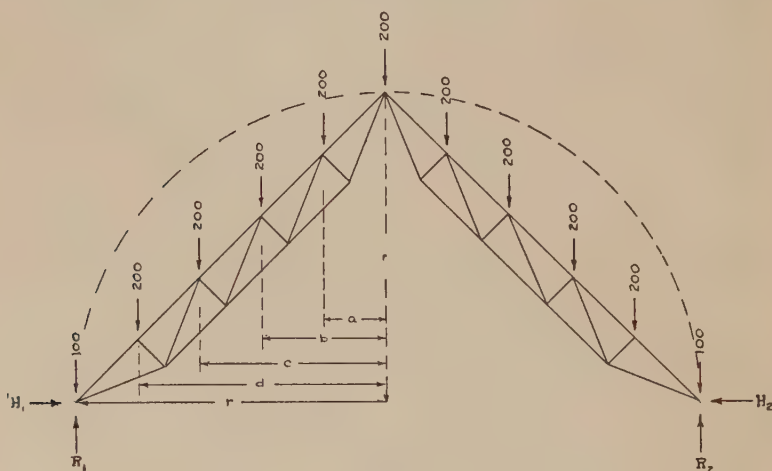


Fig. 33.

reactions illustrates a condition and a method with which the designer should be familiar.

In figure 33, the total load of 2000 pounds is symmetrical and,  $R_1 = R_2 = 1000$  lbs.

These reactions, however, balance only the vertical components of the thrust that exists in the ribs and there are still two horizontal thrusts to be accounted for. The simplest way to derive the proper value of  $H_1$  and  $H_2$  is to consider, say, the left-hand half of the truss and then to take

moments to find the center of gravity of the loading on that side of the truss, using the method of moments as outlined in article 16, and then, in the case here illustrated, we would get the equation:—

$$1000 \times \frac{r}{2} + H_1 \times r - R_1 \times r = 0.$$

Then, with  $r$  and  $R_1$ , known, solve for the values of  $H_1 = H_2$ .

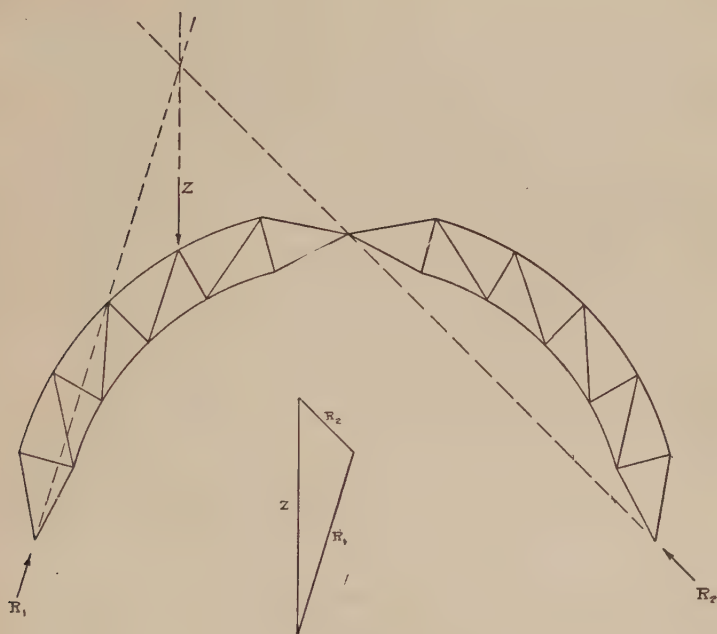


Fig. 34.

Should it be required to find the reactions for such a truss when the load was unsymmetrical and concentrated at some point  $Z$ , we would make the analysis as follows;—

The load  $Z$  is supported by the left-hand abutment and by the crown-pin, in a ratio inverse to the distance to these two points of support. Then the fraction of  $Z$  which is

supported at the abutment requires no horizontal thrust, while that fraction of  $Z$  falling on the crown-pin will require a thrust whose magnitude may be determined by resolving  $Z$  into components parallel to the axes of the ribs, and then these forces in turn, into vertical components and horizontal thrust.

Another way to determine the reactions is as follows: since part of the stress  $Z$  is transmitted to the right-hand abutment in a line through the crown-pin and that abut-



Fig. 35.

Arched or curved truss for buildings where unobstructed view and large floor space is desired. Note that the purlins are constructed like the Warren girder and are used to support the jack-rafters which, in turn, support the roof sheathing. Compare this truss with the three-hinged arch of Fig. 34.

ment, and since we know the direction of two of the external forces, we may prolong their lines of action until they meet, in order to determine the line of action of the third. See Figure 34. The horizontal component of  $R_1$  will equal the horizontal component of  $R_2$  and the sum of the vertical components of  $R_1$  and  $R_2$  will equal  $Z$ .



The line of reasoning employed in getting the value of the reactions for the three-hinged arch is similar to that employed in the analysis of the stresses and their components when we investigate a condition where the base of a ladder rests on a horizontal surface, while its upper end rests against a vertical wall, the problem in this case being to determine the amount of horizontal thrust required to prevent the ladder from slipping when it is at some specified angle and has a load on it at some specified point.

#### ARTICLE 22. SOLUTION BY REVERSAL OF THE DIAGONAL.

In the solution of problems 17 and 18 we encountered joints where there were more than two unknowns to be determined and we resorted to a method of solution which depended upon the symmetry of the loading as well as of the frame.

In the problem about to be described, (See Fig. 36), the load may or may not be symmetrical and the frame IS NOT symmetrical. The load line may be laid off and the joints *BCLA*, *CDYZ* and *ZYXA* solved in the usual manner, but when we meet with two joints each of which has three unknown members, we can proceed no farther.

In problem 21, we found that when a trapezoidal-shaped truss was symmetrical and symmetrically loaded, there was no need for any interior bracing, and that, when the load was made to be unsymmetrical, the diagonal WAS required, (See Fig. 24), and further; that if the diagonal was reversed in any quadrilateral, the change affected only the members of that quadrilateral and in some cases one of the members became "superfluous".

With this in mind, we may undertake to reverse the diagonal *VW* in the quadrilateral next to be solved and, on

inspection of the resulting frame diagram, observe that there is one member  $UV$  that becomes superfluous, thus making it possible to solve the remaining joints of the modified frame, so that finally we get a value of the stress in the member  $TA$  which is not affected in any way by any

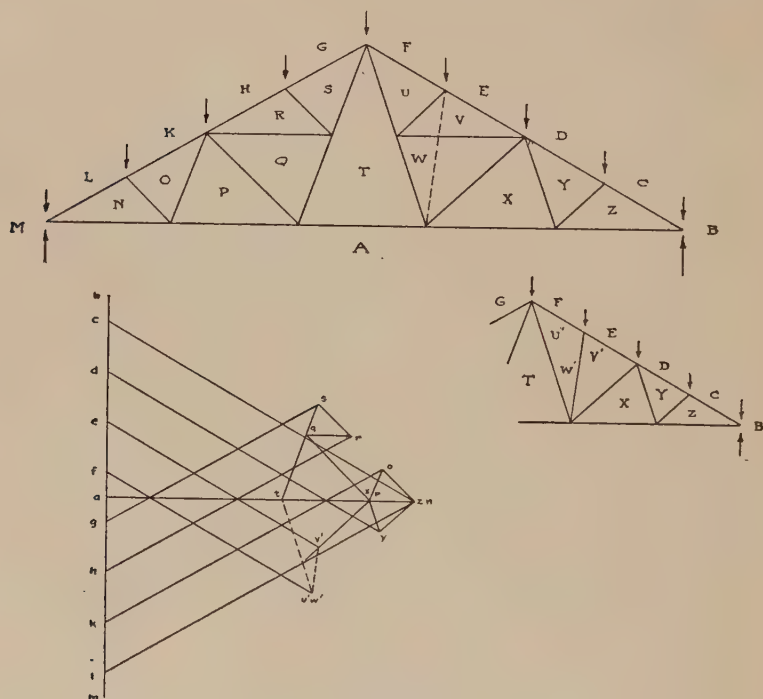


Fig. 36.

reversals that may have been made in the above mentioned quadrilateral.

With  $TA$  known and fixed, we may now replace the reversed diagonal and the member  $UV$  and make the desired determination of stress in the members in their original positions in the frame.

The method of solution by reversal of the diagonal is applicable then, to cases where, if the diagonal is reversed, one of the members of the quadrilateral in which the reversal takes place becomes equal to zero. Another case where the method is applicable is one where the reversals change the frame so that the interior becomes a symmetrical quadrilateral or trapezoid with a symmetrical load, in which the interior bracing is not required. See Fig. 24.

#### ARTICLE 23. SOLUTION BY TRIAL.

The method of solution by TRIAL, problem 34 is one not best to use unless other methods fail.

The process of solution is to proceed with the load line and solve as many joints as possible by the usual method and, then, to make a mental estimate of the value of one of following unknown stresses, and proceed, using this value until the diagram is completed or shows in some way that your estimate was in error. Then, revise your estimate and redraw the diagram without erasing the lines of the first trial. If the second trial does not close, and it probably will not do so, then the two trials will furnish you with such values as may be used in proportion to make the second and final revision of your estimate which will result in a closed diagram.

In the determination of the stresses in the more complex systems of trussing, it very often becomes necessary to make the solution of part of the structure, then to remove same, leaving its reactions to act upon the remaining portion of the structure, so as to find the effect of these loads, together with others, upon it.

In the case of the roof truss, it is sometimes possible to get the value of the stress in the lower tie by giving con-

sideration to only one half of the frame and then taking moments about the apex, involving all of the loads on that side of the truss, the reaction on that side of the truss and the required stress in the lower tie.

#### ARTICLE 24. THE HAMMER-BEAM TRUSS.

The Hammer-Beam Truss, problem 35, is one adapted to a span of from 40 to 60 feet and is commonly used in timber and steel construction for churches and other buildings where the Gothic type of architecture is to be employed and where open space and unobstructed view are desirable.

This type of truss is theoretically stable, but the stresses resulting from the loading are so proportionately high that the design is not economical unless horizontal thrust is employed to lessen the stresses in the members of the frame.

A new feature in this truss is the curved members. They are given this shape in order to make the truss pleasing to the eye, even though they cannot be constructed so economically as the straight members. In treating this problem we consider that these curved members act in a straight line connecting the two joints at the ends of that member. The design of the member itself is another matter.

In most cases this type of truss is used where elaborate decorations and refined finishes are in order and the several members are often covered with ornamental designs in wood, plaster-of-paris, terra-cotta etc. Should the member be subjected to extreme stresses, these coverings would suffer on account of the resulting deflections.

To avoid this condition it is common practice to keep the stress in the members as low as possible by means of a horizontal thrust in the side walls of the building.

In problem 29, it was observed that the providing of a certain horizontal thrust, served to relieve or reduce the stress on the lower tie members of the truss. The same condition was illustrated when we compared the stress diagrams of problems 17 and 18.

We may conclude from these examples that the more the lower tie members of any truss are raised or "cambered", the greater will be the stress in these members.

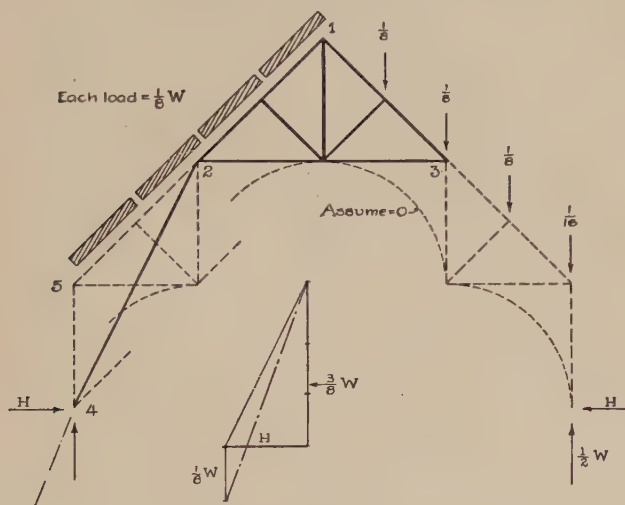


Fig. 37.

Now in this Hammer-beam truss we have an extremely high camber, so that correspondingly high stresses will result in the lower curved members unless some other means of support is used.

With the foregoing statements in mind, we will proceed to make such conditions that the curved members  $OA$  and  $VA$  are entirely free from stress when only the steady load prevails, and that the members  $NA$  and  $YA$  have their stresses reduced to a relatively low value.

To accomplish this, and still have equilibrium, we will endeavor to provide a certain amount of horizontal thrust. In order to arrive at the exact amount of thrust necessary refer to Figure 37.

This figure shows what appears like a king-post truss supported on inclined members or legs 2—4, and in order to maintain equilibrium some horizontal thrust will be necessary to prevent the trapezoidal figure from spreading at the base. The magnitude of the force  $H$  is sought, and to get it we reason as follows:—

One half of the roof load 2—1—3 or  $\frac{2}{8}$  of the total load  $W$  is being supported at the point 2, and also one half of the roof load on 5—2 or  $\frac{1}{8}$  of total load  $W$  is being supported through joint 2; the other  $\frac{1}{8} W$  lies vertically above the point of support 4 of the truss and does not require any horizontal thrust.

Then with a total load of  $\frac{3}{8} W$  acting at 2 and the direction of the support 2—4 known, we find the component  $H$ , which is the horizontal thrust necessary to maintain stability in the original truss when the stresses in the curved members  $QA$  and  $VA$  are assumed to be zero.

Now, with the value of  $H$  known, refer again to problem 35 and apply that value  $H$  in the form of a horizontal thrust, and proceed with the drawing of the load line and stress diagram in the usual manner.

The horizontal thrust is supplied in the structure by specially designed side walls. These side walls at the points opposite the trusses are modified by the addition of what are called BUTTRESSES. See Figure 38.

The side walls of brick, stone or concrete buildings are sometimes modified by the addition of PILASTERS, even though the truss does not require horizontal thrust. The



pilasters are added to the supporting walls, both for the purpose of stiffening the walls and for providing ample vertical support for the roof truss.

These pilasters may be either inside or outside the building, but when on the outside they probably have the most

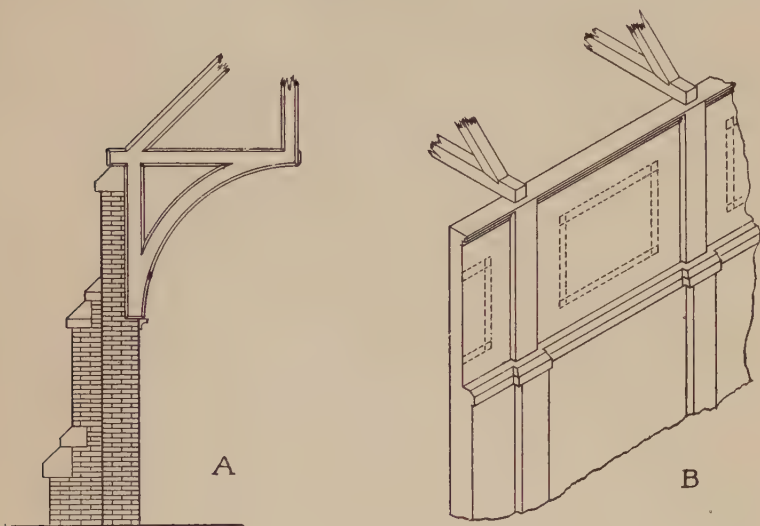


Fig. 38.

(A) Shows how the side walls of a brick or stone building may be modified by the addition of a "Buttress" so that it will be capable of taking up a certain amount of side thrust from the roof truss.

(B) Shows how the side walls of a brick or stone building may be modified by the addition of a "Pilaster" which serves to stiffen the wall and at the same time provides ample support for the roof truss and its load.

advantages to offer. The inside walls do not, as a rule, require anything in the way of columns or pilasters to give artistic effect, as other types of decoration are more desirable and plain wall space may be utilized to better advantage than one broken by the pilaster. On the other hand, unless the pilaster and other panel effects are worked into

the brick or stone outside walls of a building as it is erected, they will present a very blank or expressionless appearance, since there will be none of the high-lights or shadows which are so necessary to show relief.

Another form of the Hammer-Beam truss is illustrated in problem 36. On comparing this figure with that of problem 35, you will note that the vertical or king-rod is not present in this truss, so that it is similar to the scissor truss of problem 28.

The reason for leaving it in the form of a scissor truss is, again, to avoid excessive stresses in the several members as suggested in problem 35. Then again, examination on the diagram for problem 28 will reveal what advantages are to be gained by omitting the diagonal in the upper panel and providing for the stability of the structure by the addition of a horizontal thrust.

The loading is distributed the same as for problem 35, but the shape of the truss has been modified in such manner that the line of reasoning used to solve problem 35 will not be applicable here.

As in problem 28, we may begin solving the upper joint of the truss and work down toward the reactions in order to arrive at the amount of horizontal thrust required to maintain equilibrium in the truss. The procedure is simple for the first two joints, but at the third joint we find that there are more than two unknowns, and on examination of the adjacent joints, we find that the same conditions exist.

As in problem 31, we must now resort to the method of solution by the reversal of the diagonal and then the conditions take on the form shown in Figure 39.

The diagonals  $VW$  and  $XY$  have been reversed;  $WX$  becomes equal to zero, and the member  $CY$  becomes equal to

zero, (See Art. 13. Superfluous Members.) leaving a series of joints which are easily soluble, and at last solving the lower joint to locate the value of the reaction  $AB$ , which in turn may be resolved into its vertical and horizontal components.

With the thrust known, it will be found that the solution can now be made without further difficulty.

It will be found of interest to compare the values of the forces  $H$ , which are used to reduce stresses in these problems 35 and 36, and to make comparisons of the stresses in the

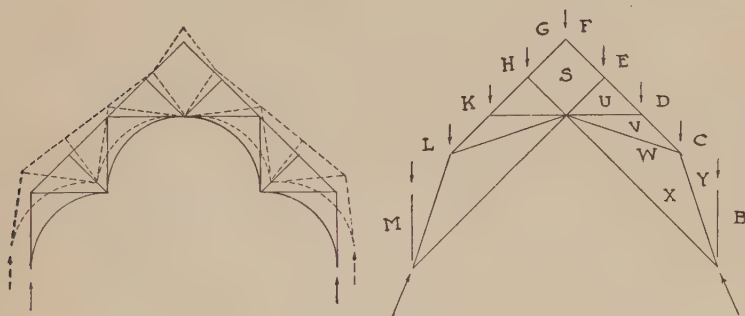


Fig. 39.

other members of the truss with those of similarly placed members in problem 36, and also to note, that in problem 36, all of the curved members are under some stress when the loading is symmetrical and under more or less additional stress when wind loading prevails.

#### ARTICLE 25. SUPPORTING BRIDGE AND ROOF TRUSSES.

There are a great many combinations which might be used in assembling the various types of building materials and it is often found desirable to support trusses or frames by means of rigid walls, piers or abutments.

When intermittent or live loads act on any of the trusses in question, there is bound to be a certain amount of deformation in the structure due to the elasticity of the several members of that structure, and also on account of the changes in temperature, which cause the spans of the bridge or roof trusses to vary considerably. It follows then, that either the supports for the ends of such trusses must be able

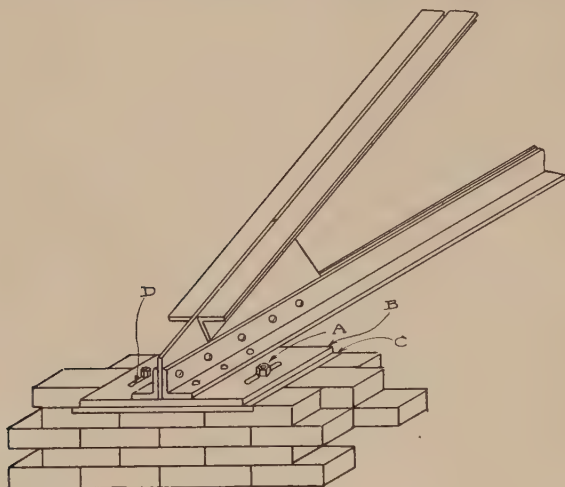


Fig. 40.

When the span is short and the roof load not large the ends of a steel truss may be anchored by the anchor bolts A passing through the slot D in the shoe B so that the end of the truss will be free to slide over the wall-plate C.

to move a corresponding amount, or, that one or both ends of the truss must be free to move on the points of support.

In short spans of steel and in wooden trusses, where the movement is relatively small, allowance for it is made when the walls of the building are of brick or stone, by having the holding-down or "anchor" bolts pass through a slot in the bearing plate or shoe fastened to the end of the truss.

This enables the truss end to make a limited movement with range enough to permit of all possible expansion and deflection movements. See Figure 40.

On large roof trusses and particularly on bridge trusses where there are vibrating and moving loads, the truss must be quite free to move or the supporting walls or piers will suffer through the continuous "racking".

To enable the desired free movement to take place, one end of the truss is supported on roller or rocker bearings, but with suitable anchor bolts to avoid the possibility of the truss rolling or sliding off the support. See Figure 41 and 42.

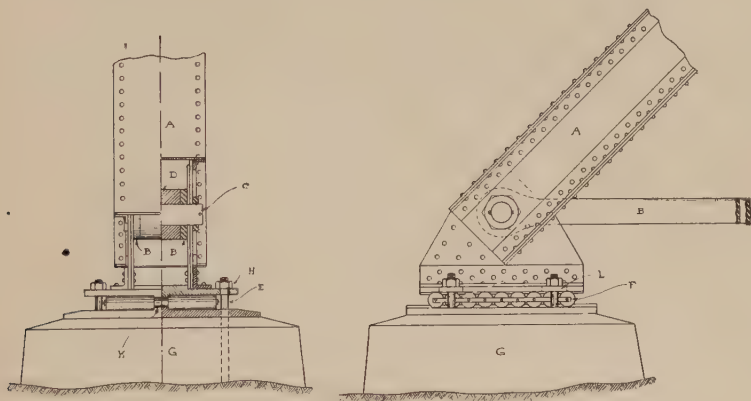


Fig. 41.

A common type of roller support under one end of a long roof truss or bridge span.

A—Compression member.

B—Tension member or lower tie.

C—Pin.

D—Washer and filler.

E—Anchor bolts bedded in the concrete pier.

F—Rollers and roller nest.

G—Foundation pier or abutment.

H—Nut on anchor bolt (Not tightened so as to allow free movement in the slot.)

K—Rib in nest and groove in rollers to prevent sidewise displacement of roller bearing on account of any possible side thrust.

L—Slot in shoe to allow limited movement of the truss on the rollers.

**Fig. 42.**

Roller bearing under one end of a bridge truss. The amount of "roll" is limited by making rollers proper diameter and spacing them so that when desired limit of roll is reached, the flattened sides will touch as shown in the figure. The bearing area is provided by supplying sufficient number of rollers.



## QUESTIONS AND PROBLEMS.

1. Name the truss of problem 17. Where is it found in use and what kind of material is generally used in its construction?
2. Compare the stress diagrams as well as the frames of problem 17 and 18 and note which members, if any are changed by cambering the truss. What is the effect on the wall reactions? Why is the truss cambered? When a series of trusses are spaced, say 10 feet apart, why do you figure that each truss must support a strip of roof 10 feet wide? How much roof does an end truss of a building support?
3. What difference would it make in the load line of problem 20 if you should begin laying it off at the point *E* instead of *B*? What, in practice, sometimes goes to make up the loads *LA* and *AM*? Are these loads necessarily equal and symmetrically placed?
4. What is true of a trapezoidal truss like problem 21 when the load is symmetrical? How does the extra load affect the stresses? How does reversing the diagonal affect the stresses? Is the steady load on *DE* equal to twice the steady load on *EF*? Why? Are the reactions equal, in cases 2 and 3? How are they found?
5. What kind of a truss does problem 22 illustrate? Where is the load placed and how is it distributed?
6. Compare problems 22 and 23. What is the name of each? Describe the differences.
7. What is the common name for the type truss shown in problem 24? What may it be used for? What kind of material may be used in its construction? Make a sketch showing how such a truss may be used to support a flat roof. What goes to make up the two kinds of loads on this truss? How may a roof of this design be drained?
8. Name the truss of problem 28. What are its advantages and its disadvantages? How may the disadvantages be overcome?

9. In problem 29, what is the stress in the member *HK*? Why? How can you manage to make the stress in the lower members *LA* and *GA* equal to any given amount?
10. Name the truss problem 35. Is it rigid and stable? What is different about the method of solution of this truss? Why is this true? How do you determine the unknown quantities? How do the curved members act? Where is this type of truss used? How and by what kind of construction are the external forces supplied for this type of truss?
11. Where is the type of truss illustrated by problem 27, used? Where is the load carried on this truss? What changes take place when the structure is moved from the closed to the open position?
12. How is the load supported on trusses such as are illustrated in problem 25 and 26? What advantages does such a truss offer?
13. What is different in the solution of problem 31? How do you decide on which diagonal to reverse? How does the reversal of the diagonal affect the stresses in the other members of the frame? What advantages are gained in reversing the diagonal?
14. How does the truss of problem 36 differ from that of problem 35? What would happen if you should begin the solution of the problem by taking the joint *ABY* first: In any case, does *QA* equal zero? How is the truss held in position? What means are used for this purpose?
15. What makes the solution of the problem 34, by the usual method, impossible?
16. What is a Buttress? What is a Pilaster? How do they differ and what are their uses in building construction?
17. How do curved members act in a roof truss? Are they efficient? Why are they used?

## CHAPTER V.

### ARTICLE 26. ACTION OF THE WIND.

The loads and forces acting upon the roof truss up to this time have been considered to be vertical, but now, dealing with the wind and its resulting pressure on an inclined surface, we will give consideration to the inclined forces.

Some authors and designers practice combining the pressure due to the wind as a vertical force with the loads of snow, roof etc., but this method of treatment, while it may give results that are practical, is obviously not correct.

It is conceded that the wind blows in a horizontal direction and that it exerts its greatest pressure when it blows in a direction normal to the surface in question. It is also conceded that it acts only on one surface of the roof or building at one time so that it imposes an unsymmetrical, inclined load on the truss.

These unsymmetrical inclined loads sometimes induce stress of opposite kind in the members of the frame than those induced by the usual vertical loads; so that it may be seen that combining the two kinds of forces might lead to serious error.

Since the plane of the roof is not normal to the direction in which the wind blows, it will not be correct to design a roof truss capable of sustaining the whole force of the wind, considered as horizontal; nor will it be correct to resolve the horizontal force into two rectangular components, one perpendicular to the roof, and the other along its surface, and then to take the perpendicular or normal component

as the one to be considered; for the pressure of the wind arises from the impact of particles of air moving with a certain velocity, and the particles not arrested are deviated from their former course so that the angles of incidence and reflection are about equal, thus giving a certain resultant whose line of action is normal to the plane of the roof. The friction of the moving body of air against the surface of the roof is small, as in the case where we consider the wind friction on the sails of a sailboat which is making use of this fact when "beating" against the wind.

#### ARTICLE 27. AMOUNT OF WIND PRESSURE ON ROOFS.

By observation and experiment it appears that, for a given velocity and pressure of the wind current on any square foot of vertical plane, the pressure against a similar plane inclined to its direction is perpendicular to its surface, and is somewhat greater than the normal component of the given horizontal pressure.

Hutton's formula for pressure of wind against any surface is one commonly accepted by architects and engineers, and while some contend that the values are larger than necessary, it is safe and may be used until we become convinced that a better one exists.

Let  $P$  = maximum pressure on a vertical plane surface in pounds per square foot.

Let  $P_n$  = maximum pressure acting perpendicular to surface of the roof in pounds per square foot.

Let  $\beta$  = Inclination of the roof.

Then—

$$P_n = P \times \sin \beta \quad 1.84 \cos \beta - 1.$$

$P$  is usually taken as 40 lbs. and is the pressure exerted on a square foot of surface normal to the wind when the velocity is about sixty miles per hour. Where the exposure is extreme, this may be raised to 50 pounds per square foot.

The table on page 90 gives the values of the wind pressure per square foot of roof at different slopes. It also gives the natural functions for the angles so that the lengths of rafter, reactions etc., may be computed.

## WIND PRESSURES AND NATURAL FUNCTIONS. .

| Incl. of<br>roof | Normal<br>Pressure | Sine   | Cosine | Tangent | Cotangent |
|------------------|--------------------|--------|--------|---------|-----------|
| 1°               | 1.2                | .01745 | .99985 | .01746  | 57.2900   |
| 2                | 2.2                | .03490 | .99939 | .03492  | 28.3994   |
| 3                | 3.1                | .05234 | .99863 | .05241  | 19.0811   |
| 4                | 4.1                | .06976 | .99756 | .06993  | 14.3007   |
| 5                | 5.1                | .08716 | .99619 | .08749  | 11.4301   |
| 6                | 6.0                | .10453 | .99452 | .10510  | 9.51436   |
| 7                | 6.9                | .12187 | .99255 | .12278  | 8.14435   |
| 8                | 7.8                | .13917 | .99027 | .14054  | 7.11537   |
| 9                | 8.7                | .15643 | .98769 | .15838  | 6.31375   |
| 10               | 9.6                | .17365 | .98481 | .17633  | 5.67128   |
| 11°              | 10.6               | .19081 | .98163 | .19438  | 5.14455   |
| 12               | 11.4               | .20791 | .97815 | .21256  | 4.70463   |
| 13               | 12.4               | .22495 | .97437 | .23087  | 4.33148   |
| 14               | 13.3               | .24192 | .97030 | .24933  | 4.01078   |
| 15               | 14.2               | .25882 | .96593 | .26795  | 3.73205   |
| 16               | 15.0               | .27564 | .96126 | .28675  | 3.48741   |
| 17               | 15.9               | .29237 | .95630 | .30573  | 3.27085   |
| 18               | 16.8               | .30902 | .95106 | .32492  | 3.07768   |
| 19               | 17.6               | .32557 | .94552 | .34433  | 2.90421   |
| 20               | 18.4               | .34202 | .93969 | .36397  | 2.74748   |
| 21°              | 19.3               | .35837 | .93358 | .38386  | 2.60509   |
| 22               | 20.2               | .37461 | .92718 | .40403  | 2.47509   |
| 23               | 21.0               | .39073 | .92050 | .42447  | 2.35585   |
| 24               | 21.8               | .40674 | .91355 | .44523  | 2.24604   |
| 25               | 22.6               | .42262 | .90631 | .46631  | 2.14451   |
| 26               | 23.4               | .43837 | .89879 | .48773  | 2.05030   |
| 27               | 24.2               | .45399 | .89101 | .50953  | 1.96261   |
| 28               | 25.0               | .46947 | .88295 | .53171  | 1.88073   |
| 29               | 25.7               | .48481 | .87462 | .55431  | 1.80405   |
| 31°              | 26.5               | .50000 | .86603 | .57735  | 1.73205   |
| 31               | 27.2               | .51504 | .85717 | .60086  | 1.66428   |
| 32               | 28.0               | .52992 | .84805 | .62487  | 1.60033   |
| 33               | 28.7               | .54464 | .83867 | .64941  | 1.53986   |
| 34               | 29.5               | .55919 | .82904 | .67451  | 1.48256   |
| 35               | 30.1               | .57358 | .81915 | .70021  | 1.42815   |
| 36               | 30.8               | .58779 | .80902 | .72654  | 1.37638   |
| 37               | 31.5               | .60182 | .79864 | .75355  | 1.32704   |
| 38               | 32.2               | .61566 | .78801 | .78129  | 1.27994   |
| 39               | 32.8               | .62932 | .77715 | .80978  | 1.23490   |
| 40               | 33.3               | .64279 | .76604 | .83910  | 1.19175   |
| 41°              | 33.9               | .65606 | .75471 | .86929  | 1.15037   |
| 42               | 34.5               | .66913 | .74314 | .90040  | 1.11061   |
| 43               | 35.0               | .68200 | .73135 | .93252  | 1.07237   |
| 44               | 35.6               | .69466 | .71934 | .96569  | 1.03553   |
| 45               | 36.0               | .70711 | .70711 | 1.00000 | 1.00000   |
| 46               | 36.5               | .71934 | .69466 | 1.03553 | .96569    |
| 47               | 37.0               | .73135 | .68200 | 1.07237 | .93252    |
| 48               | 37.3               | .74314 | .66913 | 1.11061 | .90040    |
| 49               | 37.9               | .75471 | .65606 | 1.15037 | .86929    |
| 50°              | 38.1               | .76604 | .64279 | 1.19175 | .83910    |
| 51               | 38.4               | .77715 | .62932 | 1.23490 | .80978    |
| 52               | 38.7               | .78801 | .61566 | 1.27994 | .78129    |
| 53               | 38.9               | .79864 | .60182 | 1.32704 | .75355    |
| 54               | 39.2               | .80902 | .58779 | 1.37638 | .72654    |
| 55               | 39.4               | .81915 | .57358 | 1.42815 | .70021    |
| 56               | 39.6               | .82904 | .55919 | 1.48256 | .67451    |
| 57               | 39.7               | .83867 | .54464 | 1.53986 | .64941    |
| 58               | 39.8               | .84805 | .52992 | 1.60033 | .62487    |
| 59               | 39.9               | .85717 | .51504 | 1.66428 | .60086    |
| 60               | 40.0               | .86603 | .50000 | 1.73205 | .57735    |



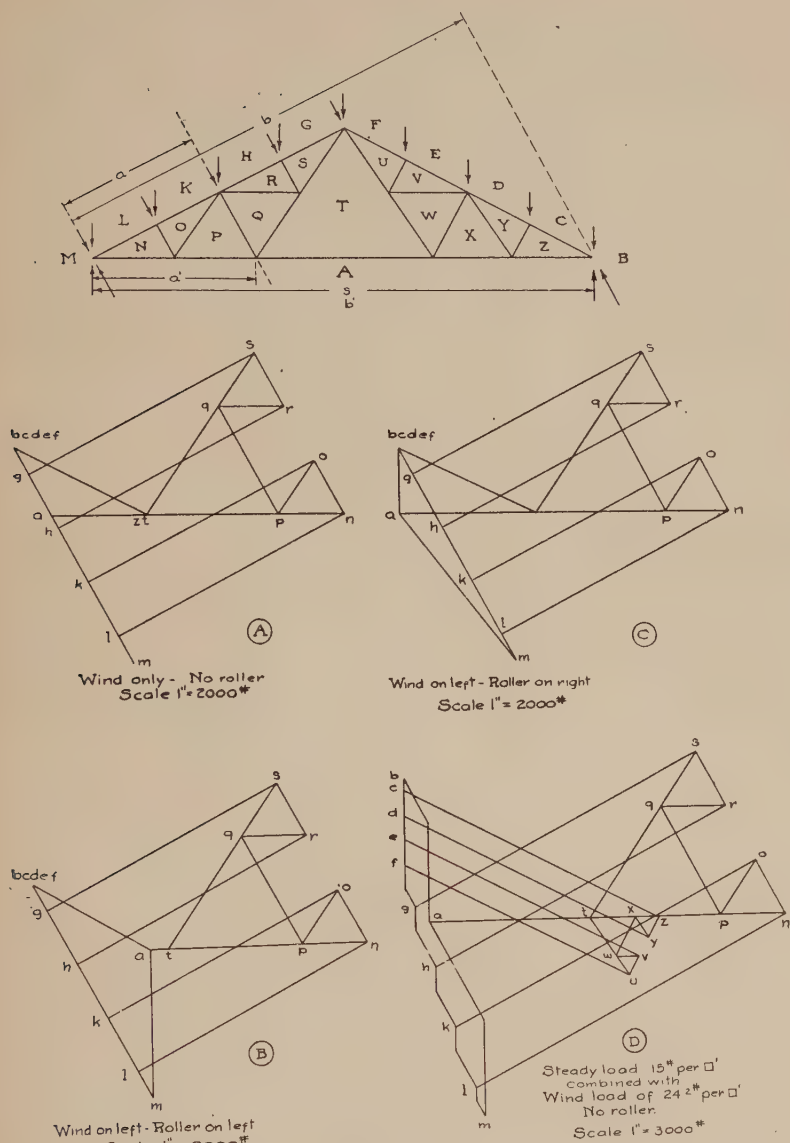


Fig. 43.

## ARTICLE 28. EFFECT OF WIND ON THE TRUSS.

It is true that it is possible to combine all external loads and forces which act upon any roof truss, and find graphically the stresses that are induced in the several members of the frame by the combined action of these loads.

If we could be sure in every case that some of the loads, like the wind load for instance, did not tend to produce stress of the opposite kind to that caused by the steady load, then the practice of combining all loads would be a safe one. This, however, is not the case, and in several types of truss the stresses caused by the wind are not of the same sign (+ or —) as those caused by the steady or snow loads.

Therefore, when we make analysis for the determination of stress in the structure it is generally better practice to find the stresses caused by the different types of loading, tabulate the several sets of results, and then to use the tabulation as a guide in designing members suitable to take care of all possible combinations of stress that may arise in any particular member.

Figure 43 is a typical type of truss, and the several stress diagrams serve to show what changes take place under different kinds of loading and when the roller bearing is placed under either end of the truss.

## ARTICLE 29. COMPUTING WIND LOAD ON A ROOF TRUSS.

Case A Figure 43, shows the stresses when the wind is acting on the left hand side of the truss and when both ends of the truss are pinned rigidly to the side walls of the building.

Slope of the roof =  $\beta = 27^\circ$ .

Span of truss =  $s = 40$  feet.

Distance between trusses = 14 feet.

Steady load = 15 pounds per square foot of roof.

Referring to the table on page 90 we find that the wind pressure per square foot of roof when the slope is  $27^\circ$  to be 24.2 pounds.

Then the total load  $W_w$  on the left hand side of the truss is:—

$$W_w = \frac{s}{2 \cos 27^\circ} \times 14 \times w = \frac{40}{2 \times \cos 27^\circ} \times 14 \times 24.2 = 7942 \text{ pounds.}$$

and the load on each of the mid-joints =

$$\frac{7942}{4} = 1985.5 \text{ pounds}$$

each, while the two end joints of this side of the truss will have  $\frac{1}{8} W$  or 992.7 pounds each.

#### ARTICLE 30. COMPUTING REACTIONS FOR WIND LOADS.

Since the load is not symmetrical in regards to the two end supports or reactions, it follows that they will not have equal values although their sum must equal the total load and their line of action is parallel to that of the wind load.

By method of moments (see Art. 16) we proceed to take the left-hand support as the center of moments and consider the forces which tend to cause rotation of the truss about this chosen center, and we find that the total wind load  $W_w$  is acting with a moment arm equal to one half the length of the rafter ( $a$ ), while the right hand reaction  $R_2$  is acting with a lever arm equal to  $b$ .

$$a = \frac{s}{2 \times \cos \beta} \quad \text{Length of rafter} = \frac{s}{2 \times \cos \beta} = l \text{ or}$$

$$a = \frac{l}{2} = \frac{s}{4 \cos \beta} \quad \text{and } b = s \times \cos \beta$$

Or, since  $a'$  and  $b'$  occupy similar positions to the sides  $a$  and  $b$  in the two similar triangles, and since they are directly proportional to  $a$  and  $b$ , we might take them as moment arms.

Then we have

$$a' = \frac{l}{2 \times \cos \beta} = \frac{s}{4 \times \cos \beta^2} \quad \text{and } b' = \text{the span } s.$$

Taking these two moment arms and forces, solve for the value of  $R_2$ .

$$R_2 = \frac{W_w \times a}{b} = \frac{W_w \times a'}{b'} = \frac{W_w \times 20}{2 \cos^2 \beta \times s} = \frac{20 \times 7942}{2 \times .891^2 \times 40} = 2500.$$

$R_1 = W_w - R_2 = 7942 - 2500 = 5442 = R_1$  Or this value of  $R_1$  may have been found by taking moments about the joint  $R_2$  and then adding the values of  $R_1$  and  $R_2$  to see if their sum equalled the total wind load.

The moment arms in most cases may be scaled from carefully constructed frame diagrams with sufficient degree of accuracy.

#### ARTICLE 31. STRESS DIAGRAM FOR WIND LOADS.

To draw the stress diagram, begin by laying off all of the determined inclined loads and their reactions so that the polygon of external forces will close, and then proceed with the solution of the joints in the usual manner until a final check on the work has been obtained.

You will note that no stress has been induced in any of the interior members of the right hand portion of the frame. This is generally true when the rafter and the lower chord or tie beam are straight.

The method of solution may involve the use of symmetry of the frame, but when we solve the right hand half of the frame we find that the interior members are superfluous so far as wind action is concerned and this fact enables us to get a value for  $TA$  and then to proceed with the solution of the left hand side of the truss.

#### Wind on Left; Roller on Left ( $B$ )

When one end of the roof truss is supported by a roller bearing, (See Figure 41) there are a few changes which take place in the stress diagram. The loading is distributed in the same manner as before, but the reactions differ. The reactions may be found by taking moments and making summation of the components of various forces, or they may be found graphically from the stress diagram for the condition where both ends of the truss are fastened to the wall as in case ( $A$ ).

Here we have an inclined reaction  $MA$  which will become a vertical reaction when a roller is used on the left-hand end of the truss, and only the vertical component of the former inclined reaction will remain at  $MA$ , since, if the horizontal force or component did exist, it could not be taken care of by the roller. Then the right-hand reaction  $R_2$  will be called upon to take up the horizontal component of  $R_1$  as well as the original inclined reactions  $R_2$ .

The resolution of the forces and the transfer of one of the components from one side of the truss to the other may be accomplished graphically by merely moving the point

"a" in a horizontal direction so that  $ma$  becomes vertical,  $m$  being already fixed on the load line.

Wind on Left; Roller on Right (C)

The stress diagram for this case is similar to the others except in that the point  $a$  is moved horizontally so that  $ab$  is vertical and the horizontal component of  $ab$  is included in the new reaction  $ma$  on the left-hand end of the frame.

The COMBINED STEADY AND WIND LOAD DIAGRAM illustrates a case, (D), where the steady load is 15 pounds per square foot of roof.

$$\text{Total steady load} = W_s = 15 \times \frac{40}{\cos 27^\circ} \times 14 = 9427 \text{ pounds.}$$

$$9427 \div 2 = 4713 \text{ pounds, the reactions.}$$

$$9427 \div 8 = 1178\frac{3}{8} \text{ pounds on each middle joint.}$$

$$9427 \div 16 = 588 \text{ pounds on end joints.}$$

With the above loads and reactions acting on the truss at the same time as the wind loads and their reactions, we may proceed with the load line by drawing each load in order as we read about the frame. This load line or polygon of external forces should close, since all loads have been balanced by corresponding wall reactions.

The solution may now be made, either by using the method of solution by symmetry or by the method of reversing the diagonal.

It will be interesting to study the relative effects of the different kinds of loading on the frame.



## ARTICLE 32. TABULATION OF RESULTS OBTAINED.

In preparing a design for any structure we must necessarily deal with forces and loads acting in any or all of the three dimensions, and, while we have become involved principally in the forces, loads and reactions acting in a single plane, we must not lose sight of the fact that still other forces may act on the members drawn in planes which we have been considering, and must give them due attention.

Most structures, particularly buildings, are made up of a series of trusses or bents built in planes parallel to each other and then fastened together by girts, purlins and lateral or sway bracing so that the whole becomes rigid and capable of resisting external force or load from any of the three directions or combinations thereof.

In designing any member of such a structure it is necessary that the magnitude and kind of stress in each of those members be known.

This information can best be secured by making stress diagrams in each of the three planes, tabulating the several sets of results and then selecting certain possible combinations, determining the maximum load or stress that each member will be called upon to support. In doing this, care must be exercised in noting whether or not the wind loads tend to reverse the kind of stress and so make it necessary for some of the members to be designed to resist tensile as well as compressive forces, and in noting the magnitude of each kind of such force.

A line diagram showing the resultant stress in all of the members of the frame is sometimes called the strain diagram; or, when a number of such diagrams are collected on one sheet, it will be called a strain sheet.

The strain sheet is the guide for the designers of the members of the frame and of the joints with which these members are made into a structure.

ARTICLE 33. LATERAL OR SWAY BRACING.

Sway bracing is necessary in frame structures in order to keep the several trusses and bents in the position of a vertical plane. The amount and kind of sway bracing varies considerably, depending upon the dimensions of the building and on the method of supporting the trusses and bents.

Figure 11 is designed to illustrate some of the different kinds of sway bracing that are in common use. The "A" type is generally called "*knee*" brace and is commonly used in both steel and wood construction. The "B" type is called the PORTAL (Article 34) and is used to hold the two parallel trusses of a bridge span rigid against the action of any horizontal thrust due to the wind or from other sources. See problem 41. The "C" type may extend over the entire panel or over only a part of it so that the bracing will not interfere with window openings etc. as indicated in this figure. The "F" type is in the plane of the ceiling of the building, while the "E" type is in the plane of the rafters of the roof truss.

It is common practice to apply sway bracing to the alternate panels of the truss or bent structure, but in many buildings, particularly those of wood, the sheathing furnishes the necessary rigidity.

The loads to be supported by the several joints of the truss are indicated by the shaded areas and their values may be computed as follows;—

$$\text{Total load } W \text{ on a central truss} = W = \frac{s}{\cos \beta} \times d \times w$$

$$\text{Load on central joint of the truss} = \frac{s d w}{8 \cos \beta}$$

$$\text{Load on end joint of the truss} = \frac{s d w}{16 \cos \beta}$$

The necessity for lateral or sway bracing may be further illustrated by referring to the case of the common bridge Fig. 44. Here is a condition where the wind action on the

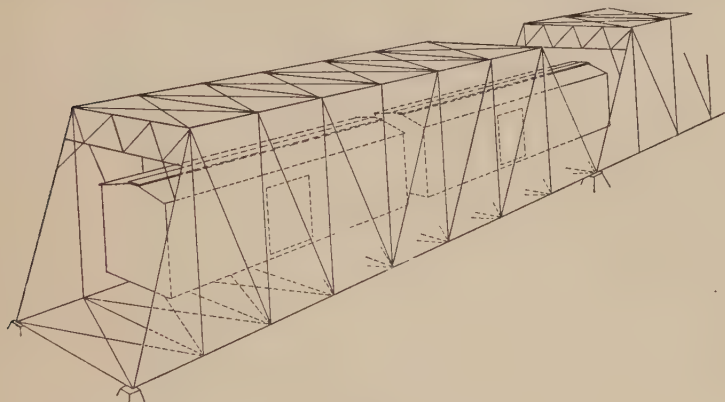


Fig. 44.

trusses alone would not amount to a great deal, but when a train of cars crossed, there would be a much larger area exposed to the action of the wind and the lateral stress in the bridge would become a considerable amount.

For example; —

If the span of the bridge were 150 feet and the height of a passenger coach or a box car 12 feet.

Then the total horizontal pressure on the deck of the span would be;—

$$150 \times 12 \times 40 = 72,000 \text{ pounds.}$$

This represents the load to be sustained by the lateral bracing in the plane of the bridge deck, since the thrust of the wind against the train is transmitted through the wheel flanges to the deck of the bridge.

#### ARTICLE 34. THE PORTAL.

The portal, problem 41, is one of the several different kinds of lateral or sway bracing in use, and its name is probably derived from its relative position when employed in bridge construction. See Fig. 25.

When the skeleton or frame of a building is made up of a series of trusses or bents resting on legs or columns, this type of sway-bracing is often used to provide lateral strength to the structure. See Fig. 11. Here the load on each end of the portal will consist of one-half the weight of the roof truss and its load, while the horizontal thrust is dependent upon the area of the end of the building and on the number of portals in the series along the side of that building.

It will be seen from Fig. 45 that we have a condition that is somewhat different from anything we have had so far. The trussing is supported on legs or columns, which, in turn, are pinned to the abutments; so that there is really a four-sided panel that would be unstable under any except vertical loads.

Since we do not have a horizontal thrust  $H$ , it follows that we must supply some other means of support in order to keep the portal in position, and this means consists of making the legs  $HL-KA$  and  $PD-AC$  continuous members with no pin joints at  $a$  or  $b$ .

When these members are made in the manner suggested, we will be introducing a rigid joint which is contrary to

one of the first assumptions made in deriving this graphical method of solution: so that, if we wish to apply the same principles for making a solution of the problem, we

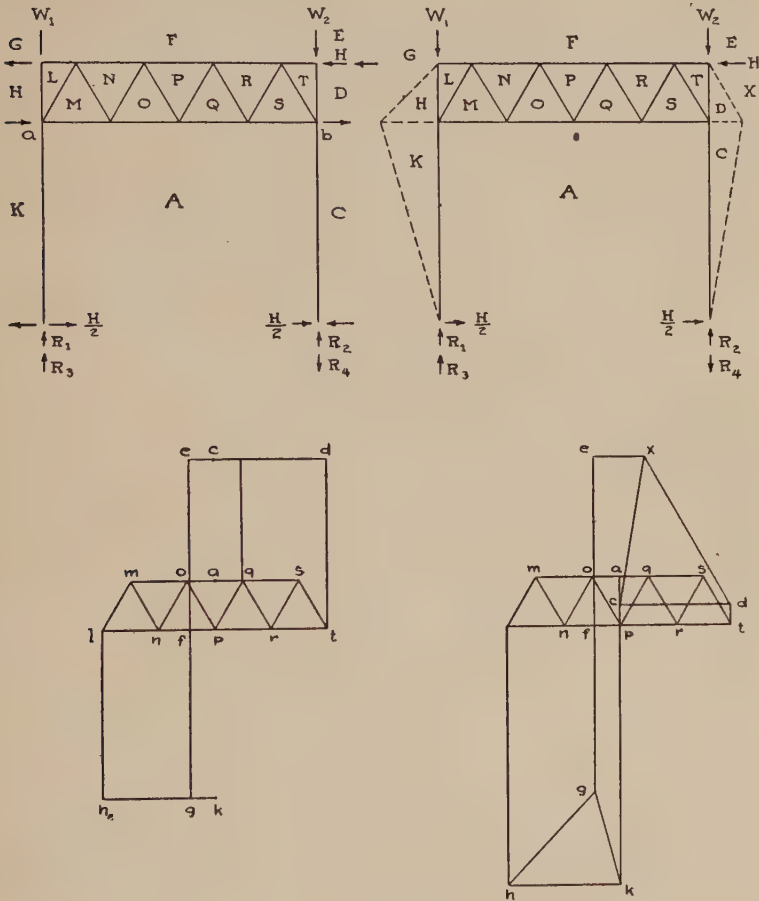


Fig. 45.

must maintain the pin-joint theory and place an imaginary couple acting at the joints of each leg, which will produce equilibrium in the leg and not disturb the equilibrium of the frame as a whole.

This couple, then, will represent the magnitude of the cross-bending stress induced in the column, in addition to the tensile or compressive stress induced by the loads and as determined by the stress diagram.

It now remains for us to determine the exact amount of this cross bending stress.

It will be readily observed that it bears a direct ratio to the horizontal thrust of any loads or forces acting on the truss. The steady loads  $W_1$  and  $W_2$  are balanced entirely by the reactions  $R_1$  and  $R_2$ , and the horizontal thrust  $H$  is balanced

horizontally by the sum of  $\frac{H}{2}$  and  $\frac{H}{2}$ ; it is balanced ver-

tically since the force  $H$  has no vertical component, but in rotation there is an unbalance that must be corrected by the reactions  $R_3$  and  $R_4$ , which will prevent a rolling action.

Now we have, at each of the abutments, a horizontal thrust which is equal to one of the forces in a couple induced in the rigid members  $KA-HL$  and  $PD-AC$ . Then, with the relative lengths of the parts  $KA$  and  $HL$  given, and with the value of one of the forces of the couple known, we can compute the value of the other two forces of the couple.

Note again that this couple does not in any way disturb the balance of the external steady or wind loads or their reactions. With the values of all force, loads and couples known, we may proceed with the drawing of the load line and stress diagram.

Instead of solving directly for the values of the couples as described above, we may resort to a method which involves the placing of several imaginary members,  $a$ ,  $b$ ,  $c$  and  $d$ ,  $e$ ,  $f$  in the positions indicated on figure 45 so to make the frame rigid, even though there are pin joints all around.



The reactions are the same as in the former case and we may proceed with the solution without difficulty. When the diagram has been completed we must find the stress, induced in the leg or column members, which will take the place of the imaginary members, and, by inspection, we can see that the bending force  $HK$  and  $DC$  will be the same as similar forces in case  $I$ , and that the other members of the bending couple may be found by taking the horizontal components of  $HG$  and  $KG$ , while their vertical components must be deducted from the stress in  $HL$  and  $KA$  respectively so as to give the true transverse stress in these members when the leg or column was rigid.

#### ARTICLE 35. ROOF TRUSSES SUPPORTED BY COLUMNS.

The type of truss shown in problem 42 and in figure 46a is very often used in the construction of large buildings such as shops, trainsheds, warehouses etc. where the span is from 80 to 100 feet or more and is supported at either end by steel columns.

If the connections of the column to the roof truss at  $a$  and  $b$  were hinged and no members, "kneebraces", were placed between the column and the lower tie of the truss, it is evident that the frame would be theoretically in equilibrium under steady and snow loads, but when a wind pressure acted on the truss either normal to the roof slope or horizontally against the side of the monitor, the columns or posts would tend to turn about their lower ends, and when the stiffness of the joints  $a$  and  $b$  was overcome, the structure would collapse.

In order to provide against such a tendency, knee braces  $c$  and  $d$  must be introduced between the lower tie of the truss and the column. These knee braces act simultaneously

when there is a tendency toward distortion of the structure, one acting in tension and the other in compression so as to counteract any such tendency.

With either tension or compression in these knee brace members, it is evident that the stress in them will be trans-

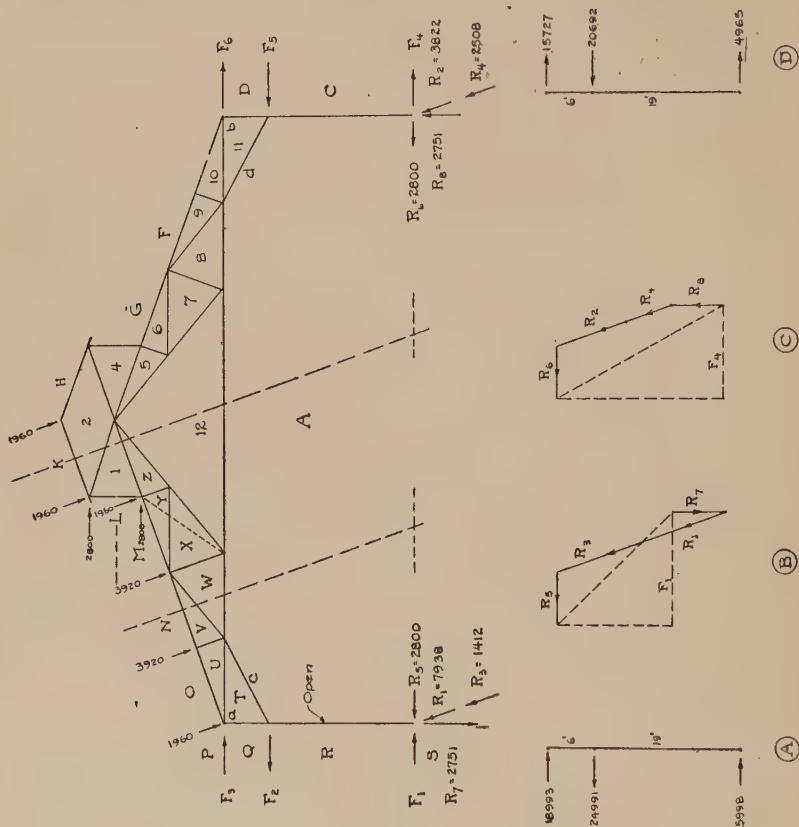


Fig. 46a.

mitted to the frame on one end and to the column on the other.

The frame stresses will suffer such readjustment as will balance this force, and the column will have to be de-

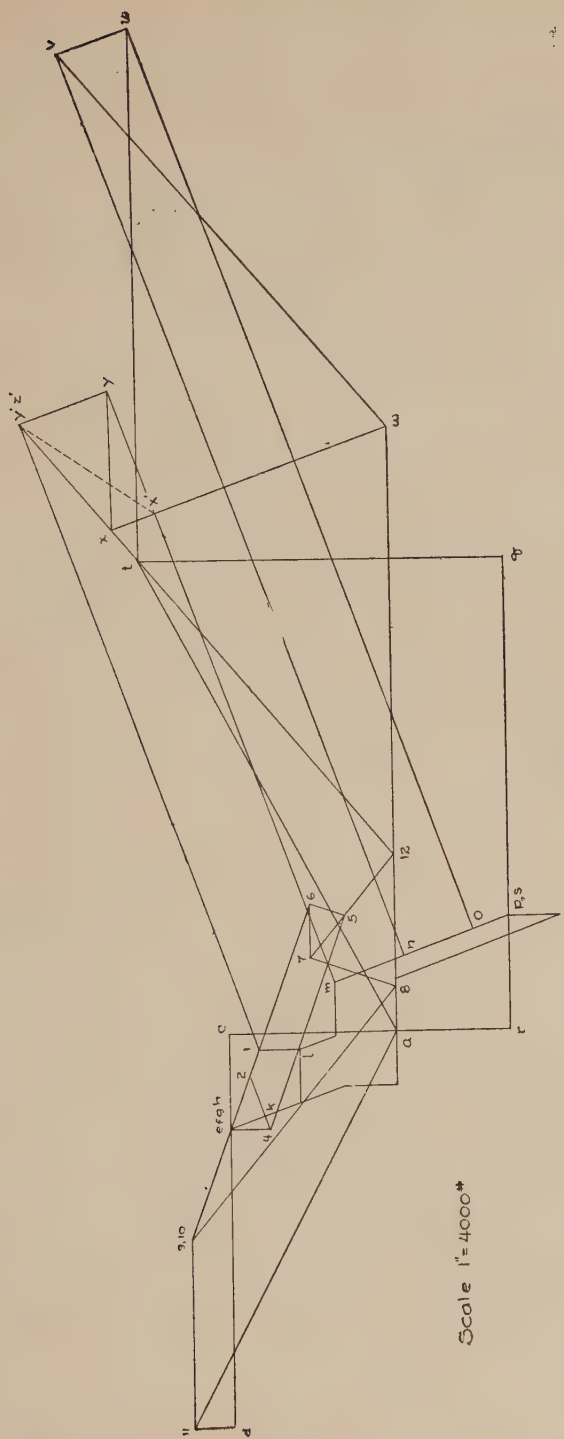


Fig. 46b.

Scale 1" = 4000"

signed to resist any crossbending which may be induced by these knee brace members.

A more detailed investigation will now be undertaken by means of calculating the loads and reactions and then drawing the stress diagram for such a truss.

In figure 46 let the span equal 80 feet.

Total length of leg or column = 25 feet.

Length of portion of column  $AM$  = 19 feet.

Slope of roof =  $\beta = 20^\circ$ .

Height of monitor = 7 feet.

Distance between trusses = 20 feet.

Wind load on left 18.4 pounds per square foot of roof.

We will assume that the sides of the building are open, as is often the case in train sheds or covered platforms which are used for transfer of freight etc. We will also assume that the sides of the monitor are glazed (windows) so that we will have a wind pressure on the vertical side as well as on the roof of the monitor.

The total normal pressure on both portions of the sloping roof will be  $\frac{40}{\cos 20^\circ} \times 20 \times 18.4 = 15676$  pounds, so that

the loads on the mid joints will be  $\frac{15676}{4} = 3920$  pounds

each and all end joints  $\frac{15676}{8} = 1960$  pounds each.

The pressure exerted on the vertical side of the monitor will be  $7 \times 20 \times 40 = 5600$  pounds or 2800 pounds at each adjacent joint.

Now the resultant of the two groups of inclined wind pressures are shown by the dash line in the frame drawing. These resultants intercept the span line and so determine the

ratio of distribution for the reactions; and if the distances from the left-hand abutment are 26.0 and 51.2 feet respectively, then the two sets of reactions may be found by taking moments about the left-hand abutment as a center as follows.—

$$R_2 = \frac{11760 \times 26.0}{\text{span}} = 3822 \text{ pounds.}$$

$$R_4 = \frac{3920 \times 51.2}{\text{span}} = 2508 \text{ pounds.}$$

$$R_1 = 11760 - R_2 = 7938 \text{ pounds.}$$

$$R_3 = 3920 - R_4 = 1412 \text{ pounds.}$$

Then,  $R_1$ ,  $R_2$ ,  $R_3$ , and  $R_4$  offer complete balance for all of the inclined wind loads, but in the case where horizontal wind loads acts against the side of the monitor, the resultant is parallel to the span line and does not intersect it. Therefore we must find the balancing reactions by taking summations of the forces and of the moments of the forces about some point.

The wind pressure of 5600 pounds tends to cause the whole truss to slide along the plane of the span so we will place

two reactions, each equal  $\frac{3920}{2}$ , one at each point of support or abutment.

This will prevent its sliding, but there still exists a tendency to roll or overturn the structure, and this must be counteracted by supplying the other two members of the couple,  $R_7$  acting downward and  $R_8$  upward. The values of  $R_7$  and  $R_8$  may be found by taking moments about either end of the truss and considering the 5600 pound horizontal wind load as acting with a lever arm of about 39.3, and balanced by either  $R_7$  or  $R_8$  with a lever arm or moment arm equal to the span.

$$\text{Then } R_7 = R_8 = \frac{5600 \times 39.3}{\text{span}} = 2751 \text{ pounds.}$$

With this couple complete; we now have all forces acting on the truss balanced, either by direct reactions, or by the remaining members of the couple; and there is complete equilibrium.

The small diagrams *B* and *C* of figure 46 show graphically in full lines how the several reactions act on each abutment and how they may be combined graphically into a resultant reaction, and how this resultant reaction may be resolved into its horizontal and vertical components. The vertical component in each case will show the amount of tension or compression in the leg or column, while the horizontal components in each case will show the extent to which the abutments resist horizontal sliding, or in other words, the bending force exerted on the end of the column.

With these horizontal components known in value, and knowing the relative lengths of the sections of the column, it will be a matter of taking moments to find the other two forces of the couple.

These couples must be included on the frame diagram so as to make the conditions the same as though there were actually a pin joint at the point where the knee braces join the columns. We may then proceed with the stress diagram as usual.

In the actual structure, the forces which balance these couples are supplied by the rigidity of the beam or column which must be designed to withstand the cross-bending as well as transverse loads.

The calculations for finding the values of  $F_1$ ,  $F_2$ ,  $F_3$ ,  $F_4$ ,  $F_5$  and  $F_6$  are as follows:—

$$F_1 = (R_1 - R_3) \sin 20^\circ + R_5 = 5998 \text{ pounds.}$$



$$F_3 = F_1 \times \frac{19}{6} = 18993 \text{ pounds.}$$

and

$$F_2 = F_1 - F_3 = 24991 \text{ pounds.}$$

$$F_4 = (R_2 + R_4) \sin 20^\circ + R_6 = 4965 \text{ pounds.}$$

$$F_6 = \frac{19}{6} \times F_4 = 15727 \text{ pounds.}$$

$$F_5 = F_4 - F_6 = 20692 \text{ pounds.}$$

The values 19 and 6 are the lengths of the two sections of the columns, or, in other words, the distance from the cross-bending loads on the column to the end supports.

All of the wind loads, reactions, and couples should be plotted to form the load line and, then, proceed with the diagram. See figure 46b. In order to complete the stress diagram it was necessary, in this case, to resort to the method of solution by the "reversal of the diagonal", since the loading on this symmetrical truss is not symmetrical.

## QUESTIONS AND PROBLEMS.

1. How does the wind act on a roof truss? How is it assumed to act? How does the pressure vary with the pitch of the roof? What is the greatest known pressure per square foot caused by the wind?
2. How do you find the value of the wall reactions when both ends of the truss are fastened to the wall? How are the reactions found when one end of the truss is fastened to the wall while the other end rests on a roller or rocker bearing?
3. Why are roller, rocker or sliding bearings sometimes used to support one end of a roof or bridge truss?
4. Is it best to make a separate stress diagram to determine the stresses in the members caused by each kind of loading or is it best to combine all loads and draw a single stress diagram which gives the stresses due to the combined loads?
5. What is the name of the type of truss in problem 38? What is a hip?
6. Why do we compute the total snow load on a truss by considering that the snow load acts on an area equal to the horizontal projection of the roof?
7. In problem 38, what is the advantage of making four different stress diagrams instead of combining them into one? How do you proceed to find the value of the reactions for this problem when there is a roller on one side of the truss?
8. How is the wind pressure on the end of the building taken care of? Does this type of load effect the stresses in the truss in any way?
9. Name some of the advantages of the curved roof?
10. What may the truss of problem 42 be used for? What

is a monitor? What is a monitor used for? Upon what parts of the roof will the wind act? Where, or at what points is the roof truss supported? How do you proceed to find the reactions for the wind that acts on the inclined portions of the roof? How do you proceed to find the reactions for the wind that acts on the vertical part of the roof? How would you treat the problem if the sides of the building were closed and the wind pressure were exerted against these sides as well as against the roof? What is the governing factor in the design of the columns or legs?

11. What is lateral bracing? What is a portal?
12. How is the snow load reckoned and at what angle of inclination of the roof may this load be neglected?
13. Prove that the reactions for the wind load on a truss which has no roller bearings are inversely as the distances from the point where the resultant of wind pressure intersects the span line.
14. Make calculation for the supporting reactions and for the couple of stress induced in the supporting columns, Figure 47A. Draw the stress diagram.

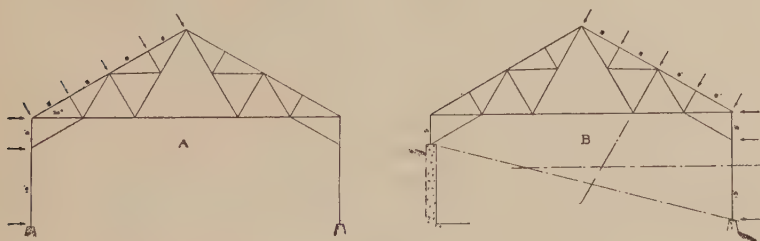


Fig. 47.

15. Make calculations for the supporting reactions and for the couples of stress induced in the supporting columns, Figure 47B. Draw the stress diagram.

## CHAPTER VI.

### ARTICLE 36. FRAMES WITH INCLINED LOADS "HEADFRAMES".

Frames having inclined loads other than wind loads are quite common. Some, like those of problem 5 and 6, are sometimes called upon to lift or support loads that are not directly beneath the end of the boom. Then, there are cases where towers are to be constructed for the support of a wire cable from which, in turn, the load is suspended, and still others where the conditions are as shown in problem 43 and in figure 48.

In Figure 48A we have the given load along the sides of the structure, the tension in the rope, and the wind pressure per square foot of surface exposed to the wind. The steady load is made up of the weight of the materials that are used in the construction; the tension in the rope is that caused by the weight to be lifted, the friction overcome and the momentum etc., giving due consideration to the slope of the skipway.

From the above we may arrive at the resultant stress on the frame and find that it does not always fall between the two points of support of the frame, but on the outside instead, indicating that the tendency of the resultant is to overturn the structure. This may seem to be an undesirable condition, but when all loads have been taken into consideration, we often find that the resultant of all will fall between the supports.

Should the resultant still fall outside the points of support, it indicates that the frame will have to be securely bolted down, "anchored", to a foundation of sufficient weight and stability to supply the needed reaction.

A careful study of the several stress diagrams will show the relative sizes of the various kinds of loads and suggest the means of making a strain sheet that will give the conditions for which we must design the several members and their joints.

It will be well to note again that, in dealing with loads and their resultant stresses, that we have been working in one plane only and that the other two planes of action must be given consideration so that when certain members of the structure appear in two of these planes, the resultant or sum of the stresses in that member will be the guide in the design of that member.

Fig. 48.—Case (A) Steady load only.

Let the height of the frame be 80 feet.

Let the width of the base be 40 feet.

Distance between bents 8 feet.

20 pounds per square foot along sides of structure.

Front:—

Then  $80 \times 8 \times 20 = 12800$  pounds on front of frame.

$$\frac{12800}{4} = 3200 \text{ pounds at } GH, HK, KL.$$

and

$$\frac{3200}{2} = 1600 \text{ pounds at } FG \text{ and } ML.$$

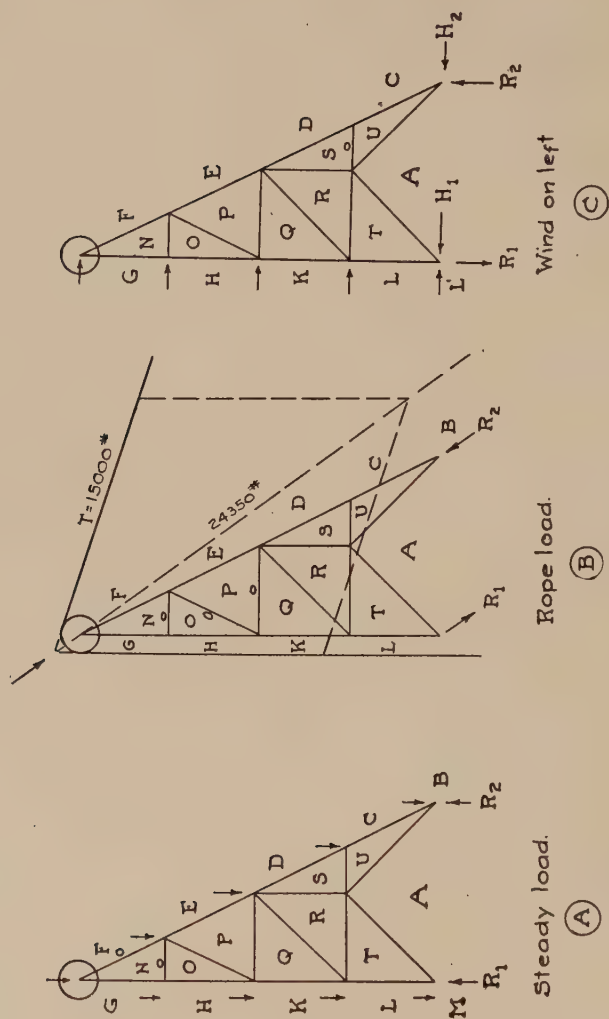


Fig. 48a.



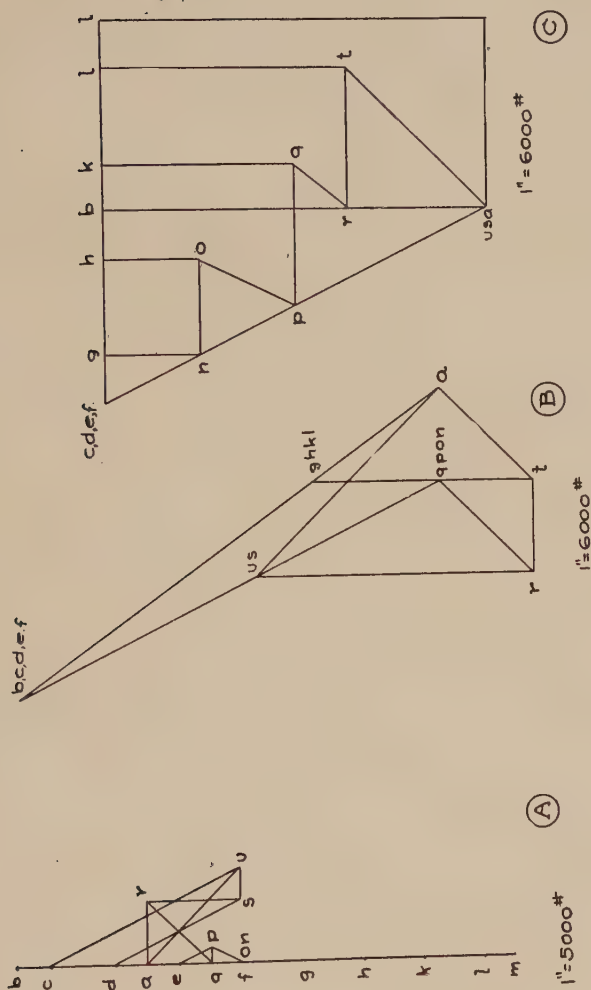


Fig. 48b.

Back:—

$$\beta = 63^{\circ}26'$$

$$\text{Then } \frac{80}{\cos 26^{\circ}34'} = \frac{80}{.894} = 89.49 = \text{length of backstay.}$$

$$89.49 \times 8 \times 20 = 14318 \text{ pounds on backstay.}$$

$$\frac{14318}{4} = 3579 \text{ pounds on } CD, DE, EF$$

and

$$\frac{3579}{2} = 1789 \text{ on } BC \text{ and } FG$$

Reactions:—

$$MA = 12800 + \frac{14318}{2} = 19959 \text{ pounds.}$$

$$AB = 7159 \text{ pounds.}$$

Case (B) Rope load only.

Tension in rope =  $W \times \sin$  of the dip single.

Tension 15000 pounds, then graphically, the resultant of the two values of  $T$  will be 24350 pounds.

Reactions:—

Taking moments about right hand support we have:—

$$R_1 = \frac{24350 \times 17.5}{40} = 10640 \text{ pounds and,}$$

$$R_2 = 34490 \text{ pounds.}$$

Case (C) Wind on left only.

Total horizontal wind pressure on front of frame equals:—

$$40 \times 80 \times 8 = 25600 \text{ younds.}$$

$$\frac{25600}{4} = 6400 \text{ pounds on } GH, HK \text{ and } KL.$$

$$\frac{25600}{8} = 3200 \text{ pounds on } FG \text{ and } LM.$$

Reactions:—

Resistance against sliding of the frame is offered by  $H_1$  and  $H_2$ , each equal to one half the wind pressure or 12,800 pounds.

Resistance against the overturning effort of the wind is found by taking moments about the left hand point of support and we have:—

$$R_2 = \frac{W \times \frac{80}{2}}{40} = 25600 \text{ pounds.}$$

$$R_1 = \frac{W \times \frac{80}{2}}{40} = 25600 \text{ pounds.}$$

Total wind load will now be balanced completely by the reactions  $R_1$ ,  $R_2$ ,  $H_1$  and  $H_2$ .

**Fig. 49.**

Typical Steel Headframe over a two-compartment vertical shaft. Kimberley skips dump the ore into a transfer car which carries it to the shipping pocket or to the stock pile. Note the difference in structural details of the various members of the frame.

## ARTICLE 37. THE SAND WHEEL.

The Sand wheel is one of the several means that may be used in elevating the tailings from a stamp mill or concentrator so that they may be carried away to a greater distance by the water or so that they may be stored in convenient form and place.

In the Lake Superior district there have been several such wheels used for this purpose and some of them are still in use. These wheels are as much as 65 feet in diameter, have round "tensil" spokes and rotate on a large hollow steel hub and shaft; the power for causing rotation is supplied by a motor driving through a pinion, which meshes with a track of gear teeth on the circumference of the wheel.

The rim of the wheel is made cylindrical, several feet wide, and is divided into a number of compartments or buckets, which, when filled at or near the bottom, serve to elevate the tailing and water to a point near the top of the wheel where they are discharged into a suitable hopper and launder and conducted to the tailings dump. It is obvious that the amount of elevation cannot be so great as the diameter of the wheel. A more detailed description of one of these wheels may be found in the Copper Handbook Vol. V p. 285 or in Scientific American Vol. 85 page 411.

Water wheels of the impulse type are often built like the sand wheel just described, when the head of water is great and the R. P. M. of the shaft is required to be low.

Water wheels of the Breast, Undershot and Overshot types, though no longer in common use, have been built on the same principle.

The analysis of the stresses in such a wheel as described above will now be undertaken, but we must first make certain assumptions and reservations as follows:—

- (1) That the spokes of such a wheel shall be either in tension or have no stress on them. They are generally made of round steel bars or rods with upset ends and with suitable joint connections at each end.
- (2) That the rim shall be made up of a series of rigid segments and capable of withstanding compressive stress.
- (3) That the joints be of the “pin” variety. This is quite true, since the spokes are limber or flexible and do not offer much resistance to bending about the joint.
- (4) That the spokes are radially located. (It most of the wheels where tensil spokes are in use, the spokes are made to cross each other near the hub in order to prevent the hub from rolling within the center of the wheel.)
- (5) The spoke which is vertical and above the hub is not able to carry a vertical compressive stress and becomes equal to zero, the loads being carried by the compression of the rim segments instead.
- (6) That the buckets, instead of being in a continous series about the inside of the rim of the wheel, are concentrated at the joints.

All of these modifications must be taken into account when desinging such a wheel, but, nevertheless, the graphical solution here outlined is a most important guide for the designer of such a wheel.

In figure 50, let  $BC$ ,  $CD$ , and  $DE$  be the loaded buckets and  $EF$ ,  $FG$  and  $AB$  the empty baskets. The power, for driving the wheel in a clockwise direction, and, which



elevates the load, is supplied by the motor driven pinion  $G$  in the direction  $GA$  "tangentially". The wheel and its load is supported entirely by the shaft, which, in turn, is supported by the inclined reaction  $R$ .

The magnitude of the driving force  $GA$  may be determined by taking moments about the center of the wheel.

In attempting to lay off the load line in the usual manner, we encounter difficulty in providing proper notation for

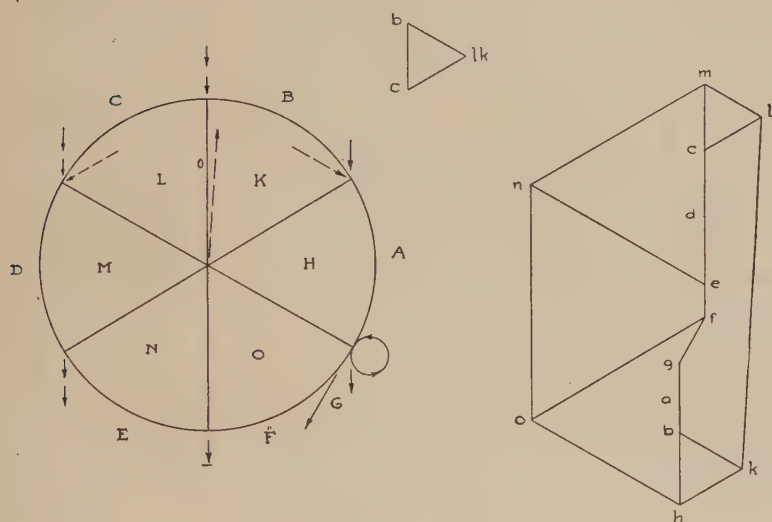


Fig. 50.

the reaction  $R$  that will avoid ambiguity. This may be accomplished by making an independent graphical solution of the upper joint  $BCLK$  where there are three unknowns  $CL$ ,  $LK$  and  $KB$ ;  $LK$ , being the spoke, must be either in tension or have no stress in it.

When the spoke is in the position shown by the frame diagram, it must be that there is no stress in it (Assumption 5). It is now possible to solve the joint graphically and

find the values of the stresses in the members  $CL$  and  $BK$ . Then, if we remove this upper segment of the wheel, and replace the values of the stresses in the members  $CL$  and  $BK$ , the reaction will be designated as  $KL$ , the spoke  $KL$  having been found to be equal to zero.

The reaction at the hub will have one component equal and opposite to the sum of all of the loads, and another equal and opposite to the driving force  $GH$ .

The load line and stress diagram may now be drawn without further modification.

#### ARTICLE 38. SHEAVE WHEEL.

Sheave wheels, sometimes called bicycle sheave wheels, are constructed in very much the same way as the sand wheel just described. The spokes are usually made of round steel rods with upset ends which are cast into the hub and rim of the wheel.

The loading on such a wheel, however, is very different than that on the sand wheel, and depends, not only upon the tension in the rope which passes over it, but also on the amount of circumference that is in contact with the rope.

The method of deriving the amount of pressure upon any part of the circumference of such a wheel is similar to that for the case where we wish to find the pressure of a belt on a driving pulley to determine the amount of power that may be transmitted to the belt for a given size pulley and for a given coefficient of friction.

Referring to figure 51, the letter  $T$  represents the tension in the rope and it also shows that this rope is acting on only a fraction of the total rim surface of the wheel.

In order to solve for the stresses that result from the two forces " $T$ ", it is first necessary to determine the value and location of the pressure caused by these forces.

Let  $p$  = pressure on unit arc.

Let  $T$  = tension in the rope.

Let  $2a$  = arc on which the rope lies.

Let  $r$  = radius of sheave.

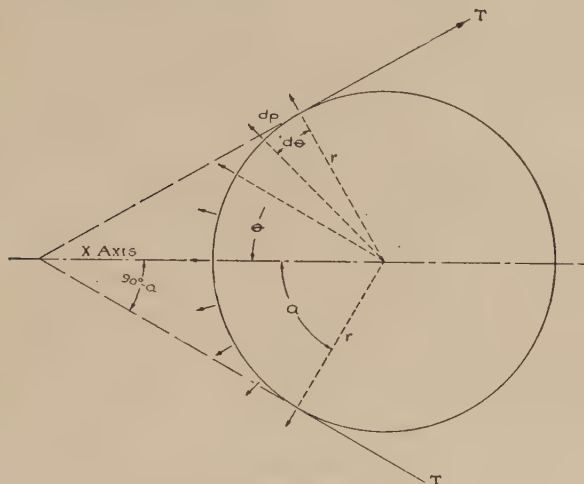


Fig. 51.

Let  $\Theta$  = the angle of any normal pressure to  $X$  axis taken through the center of the sheave and rope produced.

Normal pressure on element of arc,  $ds$ , =  $p ds$ , and its  $X$  components is =  $p ds \cos \Theta = pr \cos \Theta d\Theta$ .

$$(ds = r d\Theta)$$

Component of  $T$  parallel to  $X$  =  $T \cos (90^\circ - a) = T \sin a$ .

For equilibrium, summation  $X = 0$ , whence,

$$pr \int_{-a}^{+a} \cos \Theta d\Theta - 2T \sin a = 0$$

(Integral of  $\int \cos \Theta d\Theta = \sin \Theta$ )

$$pr \begin{bmatrix} +a \\ -a \end{bmatrix} \sin \theta = 2T \sin a$$

$$pr [\sin a - (-\sin a)] = 2T \sin a.$$

$$pr = T \quad \therefore p = \frac{T}{r}$$

From the above it appears that ( $p$ ) the pressure on a unit or increment of arc, is equal to ( $T$ ) the tension in the rope divided by ( $r$ ) the radius of the wheel. Then, to find the pressure ( $P$ ) on  $\frac{1}{8}$  circumference of the wheel, express

$\frac{1}{8}$  the circumference as  $\frac{2\pi r}{8}$  or  $\frac{\pi r}{4}$  and multiply into

the formula  $p = \frac{T}{r}$

$$P = \frac{\pi r}{4} \times \frac{T}{r} \text{ or } \frac{\pi T}{4} = \text{Pressure on } \frac{1}{8} \text{ of a}$$

circumference.

This much load then acts on  $\frac{1}{8}$  circumference and should then be properly represented on the load line by a curve, but, on account of the difficulties in drawing curves accurately and of fixed lengths and also of the fact that not much error is involved, the force is divided between the two adjacent joints and is represented as acting normal to that portion of the circumference.

The total length of the resultant will therefore be shorter than that derived directly as the resultant of the forces ( $T$ ), but it should still remain parallel to the bisector of the angle between the ropes.

The measured values approach the true values as the number of the spokes in the wheel is increased.

## ARTICLE 39. WINDING DRUMS.

Principles very similar to those outlined in connection with the sheave wheel may be used in the design of winding or hoisting drums. In this case an enormous pressure or squeeze is exerted when the rope is wound around the drum, sometimes fifty times or more.

Giving due consideration to the fact that the load or squeeze is applied gradually and at one end of the drum, it is obvious that some form of lateral bracing in its interior is essential.

Some winding drums are constructed of steel and have tensile spokes, but the cast iron drum with a cast spider and tensile lateral interior bracing is probably the most common.

## ARTICLE 40. TRUSSES WITH INTERIOR LOADING.

In the design of many structures it is desirable to provide for loading on the interior or within the panels of the truss as well as loading on the outside, with which we have been dealing in most of the foregoing problems. This type of loading consists of, in the case of the headframe, the crusher, bins, motors, engines, picking tables, grizzlies, line shafting and other heavy accessories.

In case of an extremely heavy load, such as in rock or ore bins, it is quite common practice to design the bin and headframe separately as two distinct structures and, in this manner, avoid trouble due to unequal settling or from deflections which may be caused by the subsequent loading or by relatively unequal settling of foundation supports.

In other cases, such as the support of crushers above the bins, line shafting, skip-dumps etc., it may be necessary to design the truss or frame so that it will carry these loads.

If we proceed to follow the usual methods of solution we will find that it is difficult to represent the interior loads on the load line.

To avoid this difficulty, we may have to dismember the structure, or at least divide it into certain units and then to analyze these units independently, beginning at the top of the structure and working downward toward the foundation of the whole. Each time a unit has been analyzed we use the supporting reactions of the solved unit as loads on the next unit below and supporting the upper one. In this way it will be possible to represent the interior loads in the structure and to find the relative stress induced by them in the frame.

The above outlined principle was first used in the solution of the sand wheel, article 37.

#### ARTICLE 41. GRAPHICAL ANALYSIS OF STRESS IN JOINTS.

In preparing the design of any frame structure, we become interested 1st in the requirements or loading which that structure is expected to support; 2nd in the magnitude and kind of stress induced in each of the members of the structure by the loads on it; 3rd in the type and size of material which goes to make up the members of the structure; 4th in the details of construction of the joints required to fasten the different members together in a strong and efficient manner.

In the design of these joints, we refer to the stress diagram of the structure as a whole, and find from it the number of members meeting at one joint, the nature and magnitude of each of these stresses which meet at the common point called the joint, and then proceed to make further



analysis and resolutions of forces in order to learn the effect on the several parts of the joint in question.

There are a great many types of structural joints, but the few general principles involved in their analysis are illustrated by the analysis of the simple joint shown in figure 52 *A, B, C, D*.

In figure *A*, we have known, the rafter compression  $R$  and the lower tie tension  $T$  from the stress diagram proper. The vertical stress in the supporting column is the same as the supporting reaction given on the stress diagram. In

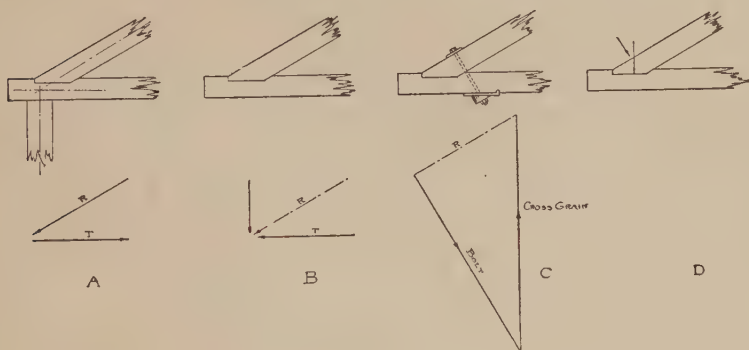


Fig. 52.

figure 52 *B*, we have resolved the rafter compression into a horizontal component  $T$ , which will be the amount of stress required to prevent the rafter from sliding off from the lower tie-beam, and which must in this case be provided by the "with-the-grain" shear in the fiber of the tie-beam. The vertical component represents the "cross-the-grain" compression in the fibers of the lower tie-beam and the angular-grain compression on the end of the rafter.

In figure 52 *C*, we have introduced a bolt to take up the horizontal thrust of the rafter member, and on the stress diagram we have shown the vertical or cross-the-grain

compression on the lower tie and another component parallel to the centerline of the bolt which represents the tension in the bolt as well as the cross-the-grain compression on the fiber under the washer which is placed under the head of the bolt.

The end-grain compression caused by the set-in lug on the lower side of the tie-beam consists of the horizontal component of the stress in the bolt, or in other words, the value " $T$ ". It will be noted that there is no dependence placed on the shearing value of the bolt in the wood.

#### QUESTIONS AND PROBLEMS.

1. In problem 43, how do you determine the tension in the rope when the load is at rest? How would you proceed to find the tension in the rope if the load were moving (being hoisted)? Will this induce a greater or a less stress in the rope? What would happen if the skip became caught or jammed in the shaft? Why is the diagonal bracing arranged in pairs?
2. What are the new features in the solution of the sand wheel problem?
3. How does the sheave wheel problem 46 differ from the sand wheel problem?
4. How could you find the pressure per segment of wheel in problem 46 if there were 12 spokes instead of 8?
5. What kind of a structure does problem 47 represent? Why are the loads on the joints 1, 2, 3 and 4 greater than the loads on 6, 7, 8 and 9? Why is the load on 5 the greatest of them all? How do you find the values of the reactions, both in magnitude and direction? What new features are shown in this problem?

# APPENDIX A.

0.

## GENERAL INSTRUCTIONS

LETTER BOTH DRAWINGS OF FRAMES AND STRESS DIAGRAM.

MARK THE VALUE AND KIND OF STRESS ON THE MEMBERS OF THE FRAMES, INCLUDING REACTIONS.

+ = COMPRESSION.

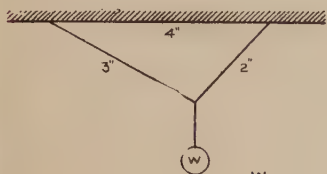
- = TENSION.

ANSWERS TO PROBLEMS HAVING MORE THAN ONE SET OF RESULTS ARE TO BE TABULATED AS PER FORM GIVEN WITH THAT PROBLEM.

NUMBER EACH PROBLEM AND INDICATE THE SCALE USED, BOTH FOR THE FRAME AND THE STRESS DIAGRAM.

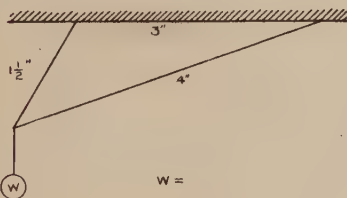
GIVE THE DATA FOR EACH PROBLEM (LOAD, SPAN, etc.).

1.



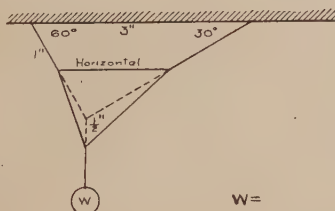
W =

2.



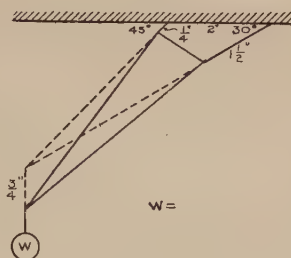
W =

3.



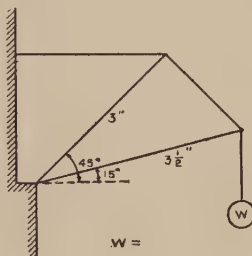
W =

4.



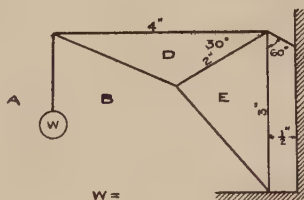
W =

5.



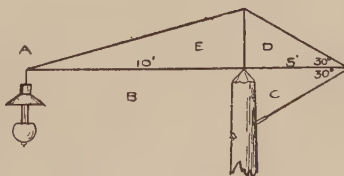
W =

6.



W =

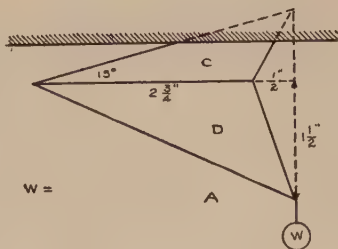
7.



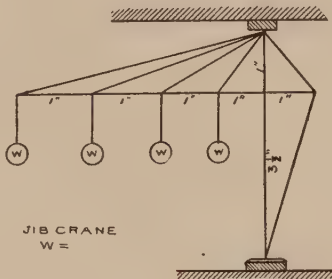
W =

# APPENDIX A.

8.

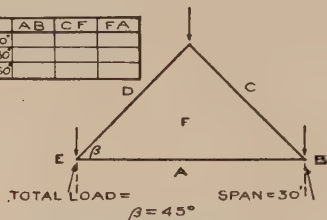


9.



10.

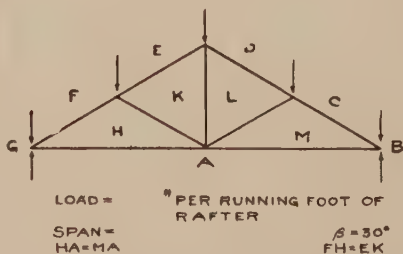
|     | AB | CF | FA |
|-----|----|----|----|
| 10° |    |    |    |
| 30° |    |    |    |
| 60° |    |    |    |



CHANGE ANGLE OF REACTIONS FROM 10° TO 30° TO 60°. NOTE DIFFERENCE IN STRESSES.

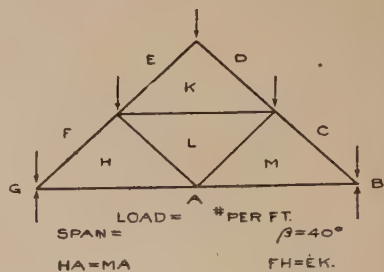
11.

— KING POST TRUSS —

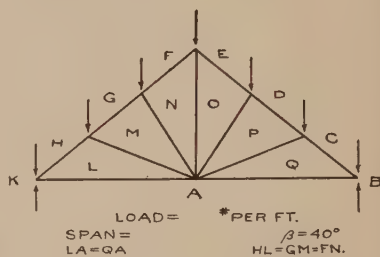


12.

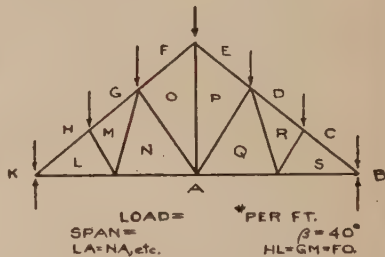
— QUEEN POST TRUSS —



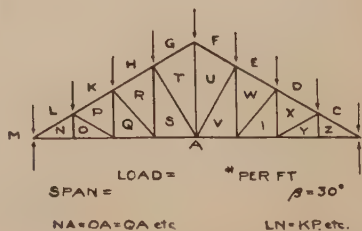
13.



14.

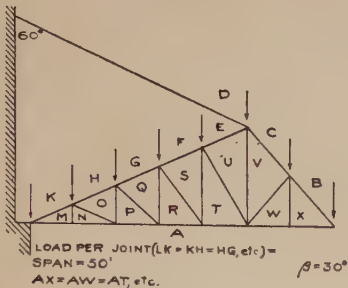


15.

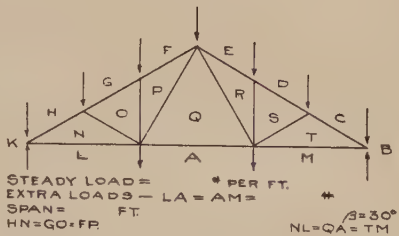


## APPENDIX A.

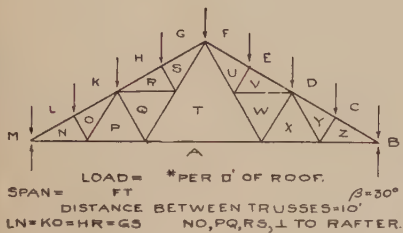
16.



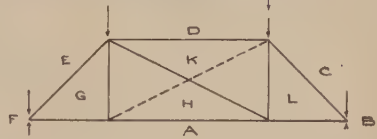
20.



17.



21.

[illegible]

18.

— FINK TRUSS —



SAME AS # 16 CAMBER = FT.  
NOTE EFFECT OF CAMBER.

22.

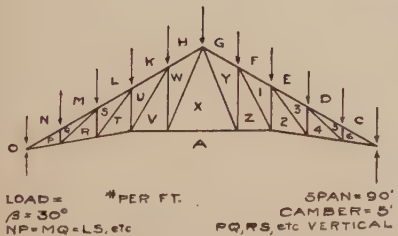
— PRATT BRIDGE TRUSS —



LOAD = # PER FT. SPAN = FT.  
EXTRA LOAD ON ED = #  
 $\beta = 60^\circ$  LH = NG = PF, etc.

Put diagonals in panels MN, OP, QR, and ST so that each will be in tension.

**19.**



23.

— HOWE BRIDGE TRUSS —



LOAD = \* PER FT. SPAN = FT  
EXTRA LOAD ON ED = \*  
 $\beta = 60^\circ$  LH = NG = PF, etc.

Put diagonals in panels MN, OP, QR, ST so that each will be in compression.

# APPENDIX A.

24.

## — WARREN GIRDER —

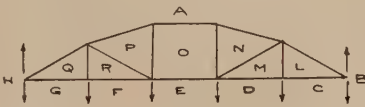


ROOF LOAD = \* PER FT. SPAN = FT.  
CEILING " = " " "  $\beta = 60^\circ$

Panels are equal equilateral triangles.

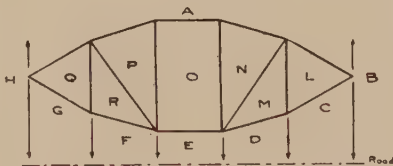
25.

## — BOWSTRING BRIDGE —



LOAD = \* PER FT. SPAN = 50'  
 $\beta = 30^\circ$  HEIGHT = 8'  
GQ = RF = OE, etc. RP  $30^\circ$  to QR, PO 1.

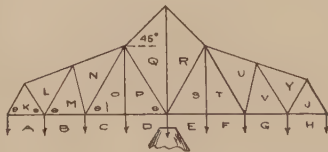
26.



TOTAL HEIGHT = 16'

Lower member curved, otherwise same as #28.

27.



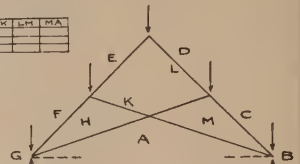
## — DRAW BRIDGE TRUSS —

LOAD = 200 \* PER FT. SPAN = FT.  
AK = MB = OC, etc.  $\theta = 60^\circ$

28.

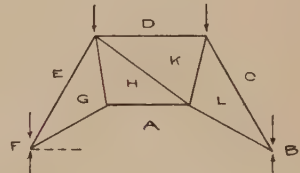
## — SCISSOR TRUSS —

| No. | AB | CD | DE | EF | FG | GH | HA |
|-----|----|----|----|----|----|----|----|
| 1   |    |    |    |    |    |    |    |
| 2   |    |    |    |    |    |    |    |
| 3   |    |    |    |    |    |    |    |



LOAD = \* PER FT. SPAN = FT.  
 $\beta = 45^\circ$  FH = EK.  
1- Omit KL Thrust? } Separate diagrams.  
2- KL horizontal  
3- KL vertical

29.

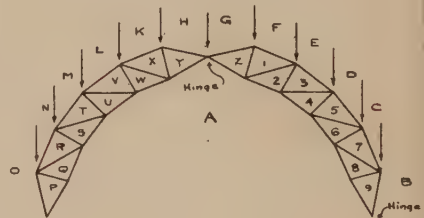


LOAD = \* PER FT. SPAN = 40 FT. HEIGHT = 18'  $\beta = 30^\circ$  &  $60^\circ$   
CAMBER = 8'

Greatest allowable stress in AL = (-) \*  
What will the wall thrust be?

30.

## — THREEHINGED ARCH —



SPAN = 2R  
PO, RS, TU etc are RADII and are each =  $\frac{1}{12}$  Semi-circumference of hinge circle.

1. DRAW DIAGRAM WHEN THERE IS A VERTICAL PRESSURE OF \* ON THE JOINT KL ONLY.

2. DRAW DIAGRAM FOR DEAD LOAD ~ EQUAL LOAD ON ALL JOINTS

3. DRAW DIAGRAM FOR WIND ON LEFT

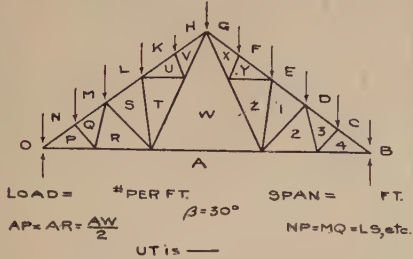
~TABULATE RESULTS.~



## APPENDIX A.

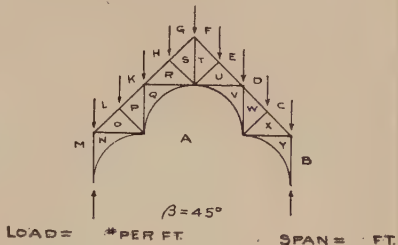
31.

— REVERSAL OF DIAGONAL —

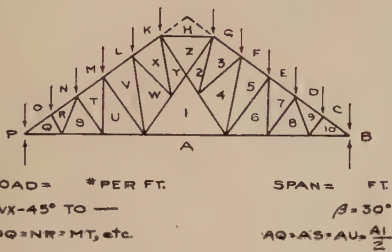


35.

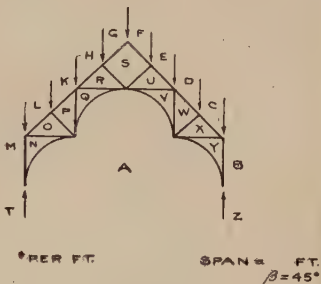
— HAMMER BEAM TRUSS —



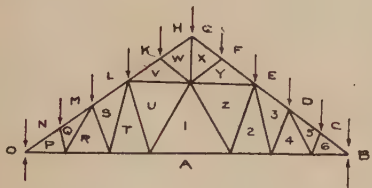
32.



36.



33.



LOAD = # PER FT. SPAN = FT.  
 VU is — wx is |  $\beta = 30^\circ$   
 NP = MQ = LS, etc AP = AR = AT =  $\frac{AL}{2}$

37.

— ACTION OF THE WIND —

TRUSS, PROBLEM 13.

1. WIND ON LEFT - NO ROLLER.

2. " " " " ROLLER ON RIGHT.

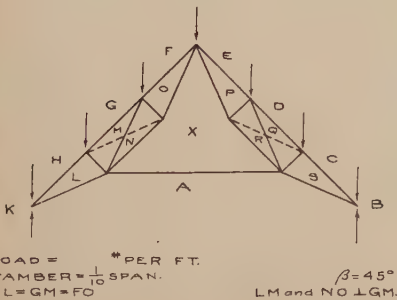
3. " " RIGHT - " " " " .

DISTANCE BETWEEN TRUSSES = 10'.

SPAN = FT.

34.

— SOLUTION BY TRIAL —



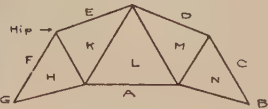
LOAD = \* PER FT.  
CAMBER =  $\frac{1}{10}$  SPAN.  
HL = GM = FO

$\beta = 45^\circ$   
LM and NO  $\perp$  GM.

38.

— MANSARD —

|     | 1 | 2 | 3 | 4 | Max | Min |
|-----|---|---|---|---|-----|-----|
| GAB |   |   |   |   |     |     |
| SUN |   |   |   |   |     |     |
| DOM |   |   |   |   |     |     |
| EST |   |   |   |   |     |     |
| NA  |   |   |   |   |     |     |
| IN  |   |   |   |   |     |     |
| HE  |   |   |   |   |     |     |
| LI  |   |   |   |   |     |     |
| KE  |   |   |   |   |     |     |



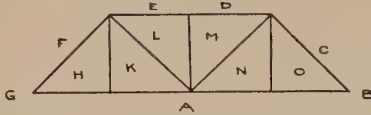
STEADY LOAD = # PER D<sup>2</sup> OF ROOF  
SNOW " = " " D<sup>2</sup> (HORIZ. PROJ.).  
DISTANCE BETWEEN TRUSSES = 12 FEET.  
SPAN = FT. HEIGHT TO HIP = 0.3 OF SPAN.  
CAMBER =  $\frac{1}{12}$  SPAN.

1. STEADY LOAD. FH~60° to —  
2. SNOW " EK~20° to —  
3. WIND ON LEFT, ROLLER ON RIGHT.  
4. " " RIGHT, " " " KH~60° to —

# APPENDIX A.

39.

TRUSSES 10' APART.

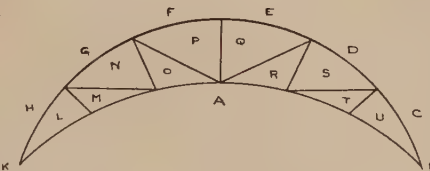


STEADY LOAD = \* per D'.  
 SNOW " = \* " (hor proj).  
 SPAN = FT.  $\beta = 45^\circ$   
 HA = KA EL = DM.

1. SNOW LOAD
  2. WIND LEFT, NO ROLLER.
  3. " " , ROLLER LEFT.
  4. " RIGHT, " "
- Each combined with steady load.

Tabulate same as Prob. 38; note order, rafters, suspenders and braces, lower ties.

40.



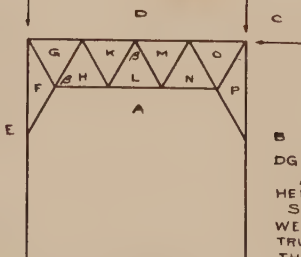
DISTANCE BETWEEN TRUSSES = 10 FT.  
 STEADY LOAD = \* PER D'.  
 SNOW " = \* PER — D'.  
 SPAN = 60' RADII = 32.5-52.6'  
 LM, NO, PQ  $\perp$  OUTER CURVE HL = GN = FP

1. STEADY LOAD.
2. SNOW "
3. WIND ON LEFT—ROLLER ON RIGHT.
4. " " RIGHT— " "
5. COMBINED (1) and (3).

|    | 1 | 2 | 3 | 4 | 5 |
|----|---|---|---|---|---|
| KA |   |   |   |   |   |
| AB |   |   |   |   |   |
| BV |   |   |   |   |   |
| DS |   |   |   |   |   |
| EO |   |   |   |   |   |
| FP |   |   |   |   |   |
| GR |   |   |   |   |   |
| HL |   |   |   |   |   |
| LT |   |   |   |   |   |
| LN |   |   |   |   |   |
| NO |   |   |   |   |   |
| OP |   |   |   |   |   |
| PS |   |   |   |   |   |
| QR |   |   |   |   |   |
| ST |   |   |   |   |   |
| TL |   |   |   |   |   |
| UA |   |   |   |   |   |
| VA |   |   |   |   |   |
| WA |   |   |   |   |   |
| XA |   |   |   |   |   |
| YA |   |   |   |   |   |
| ZA |   |   |   |   |   |

41.

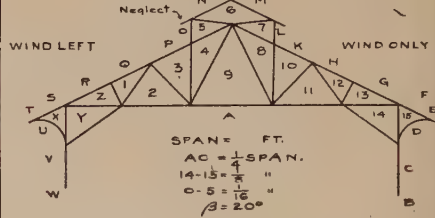
— PORTAL —



DG = DK = DM = DO  
 $\beta = 60^\circ$   
 HEIGHT = SPAN  
 WEIGHT OF ROOF TRUSS = \*  
 THRUST = BC

42.

— TRAIN SHED —

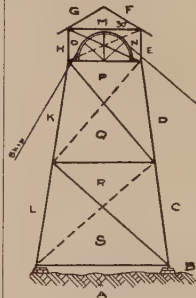


DISTANCE BETWEEN TRUSSES = 12'.

TK = RZ = QI = PS = 54.  
 NO // P3

3-4 VERTICAL  
 12  $\perp$  RZ

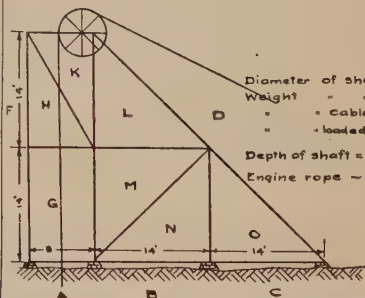
43.



WIDTH OF BASE = 20'  
 " " TOP = 10'  
 HEIGHT TO PD = 30'  
 SKIP ROPE 70' to —  
 ENGINE " 45' "

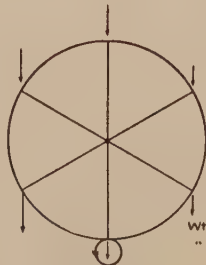
WT. OF SKIP = \*  
 " " PULLEY = \*  
 WEIGHT ALONG SIDES = \* per ft  
 ROOF LOAD = \*  
 HO = 5' RAFTER = 8'

44.



Diameter of sheave = 6'  
 Weight " = 1200 \*  
 " Cable = 1.58 per ft  
 " loaded cage = 8000 \*  
 Depth of shaft = ft  
 Engine rope ~ 30' to —

45.



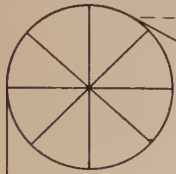
Diam. of wheel = ft  
 Wt of rim = \* per ft.  
 Neglect weight of spokes.  
 Wt of loaded buckets = \* each  
 " " empty " = \* each.

~ SAND WHEEL ~  
 Tensile Spokes

# APPENDIX A.

46.

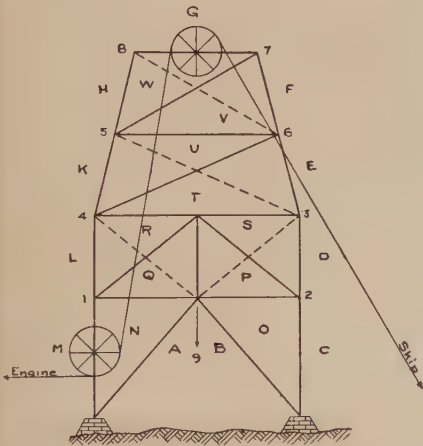
~ SHEAVE WHEEL ~



Diam of sheaves = 12 ft.  
T =

T# Tensile spokes

47.



DIAMETER OF SHEAVES = 6 FT.

WIDTH AT BASE = 25 FT.

" " TOP = 15 FT.

HEIGHT TO 1-2 = 15 "

" " 3-4 = 25 "

" " 5-6 = 35 "

" " 7-8 = 45 "

WT OF SKIP = 4A

" " PULLEY = A

" " ROPE = 8A

" " AT 1; 2; 3; 4

" " = 5A EACH

WT AT 9 = 20A

" " 5; 6; 7; 8

" " = A EACH.

SKIP ROPE IS 60° TO HORIZONTAL.

ENGINE ROPE IS HORIZONTAL.



# INDEX.

|                               | Page                              |
|-------------------------------|-----------------------------------|
| Abutments .....               | 82                                |
| Appendix .....                | 129, 130, 131, 132, 133, 134, 135 |
| Arch Truss .....              | 72                                |
| Bearings .....                | 82, 83, 84, 85                    |
| Bridge Trusses                |                                   |
| Bowstring .....               | 61                                |
| "K" Type .....                | 62                                |
| Howe .....                    | 63                                |
| Pratt .....                   | 63                                |
| Swingbridge .....             | 65                                |
| Buttresses .....              | 79                                |
| Couples in rigid column ..... | 106                               |
| Column Supports .....         | 103                               |
| Definitions of terms .....    | 10, 11, 12                        |
| Derricks .....                | 21, 23                            |
| Drawbridges .....             | 65, 66                            |
| Drawing Instructions .....    | 15                                |
| Engineer's Scale .....        | 16, 17                            |
| Expansion of Trusses .....    | 82, 83, 84                        |
| Force                         |                                   |
| Representation of .....       | 13                                |
| Resultant of .....            | 13                                |
| Triangle of .....             | 14                                |
| External .....                | 17                                |
| Fink Truss .....              | 49                                |
| General Instructions .....    | 15                                |
| General Principles .....      | 6, 22, 24                         |
| Hammer-beam Truss .....       | 76, 77, 78, 80, 81                |
| Headframes .....              | 112, 118                          |
| Stress Diagram .....          | 114                               |
| Frame Diagram .....           | 115                               |
| Loads on .....                | 113, 116                          |
| Reactions .....               | 117                               |
| Interior Loading .....        | 52, 53, 54, 125, 126              |
| Jib Cranes .....              | 26, 27                            |
| Joints .....                  | 126, 127                          |
| King-post Trusses .....       | 42, 43                            |

|                                         |                               |
|-----------------------------------------|-------------------------------|
| Lateral Bracing of Bridge Trusses ..... | 99                            |
| of Structures .....                     | 98                            |
| Loads on Roof Trusses .....             | 33, 34, 35                    |
| Distribution of .....                   | 35                            |
| Computing of .....                      | 35                            |
| Moments, Theory of .....                | 55, 56, 57                    |
| Notation, System of .....               | 18                            |
| Nature of Stress .....                  | 25                            |
| Outline of methods .....                | 12                            |
| Piers .....                             | 81                            |
| Pilasters .....                         | 79                            |
| Portal .....                            | 100, 101, 102                 |
| Pratt Truss .....                       | 58, 59                        |
| Preface .....                           | 3, 4, 5                       |
| Questions and Problems .....            | 28, 48, 85, 86, 110, 111, 128 |
| Roller bearings under trusses .....     | 95                            |
| Roof Trusses .....                      | 29, 30                        |
| Types of .....                          | 30                            |
| Simple .....                            | 39, 40                        |
| Roofs .....                             | 31                            |
| Types of .....                          | 32, 33                        |
| Sand Wheel .....                        | 119, 120, 121                 |
| Scissor Truss .....                     | 66, 67, 68                    |
| Sheave Wheels .....                     | 122, 123, 124                 |
| Solution by                             |                               |
| Symmetry .....                          | 49, 51                        |
| Reversing the diagonal .....            | 73, 74, 75                    |
| Trial .....                             | 75                            |
| Stresses in the frame .....             | 20, 40                        |
| Stress, Nature of .....                 | 25                            |
| Superfluous Members .....               | 44                            |
| Suspended Roofs .....                   | 45, 46, 47                    |
| Swing Bridges .....                     | 65, 66                        |
| Symmetry, Solution by .....             | 49                            |
| Tabulation of results .....             | 97                            |
| Three-hinged Arch .....                 | 69, 70, 71                    |
| Trapezoidal Shaped Trusses .....        | 57                            |
| Train Shed Problem .....                | 103, 104                      |
| Triangle of forces .....                | 17                            |
| Two-hinged Arch .....                   | 69                            |
| Warren Girder .....                     | 60                            |
| Weights of                              |                               |
| Roof Trusses .....                      | 37                            |
| Roofing Materials .....                 | 37                            |
| Snow .....                              | 37, 38                        |



## Wind

|                               |        |
|-------------------------------|--------|
| Action of the .....           | 87     |
| Pressure, amount of .....     | 87, 88 |
| Pressure on roofs .....       | 88     |
| Table of Pressures .....      | 90     |
| Effect on Roof Trusses .....  | 92     |
| Stress diagrams .....         | 91     |
| Computing Loads .....         | 93, 94 |
| Effect on the reactions ..... | 95     |
| Winding Drums .....           | 125    |





